UG/5th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH SEC-1

(Discrete Mathematics)

Full Marks : 32 | Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(4 Marks)

1. Answer any *four* questions : $1\times4=4$

(a) Give an example of a multigraph.

(b) Show that if in a graph G there is one and only one path between every pair of vertices, then G is a tree.

(c) Find the complement of the Boolean function $f=(a+b)(a+c')$.

(d) Let A, B be subsets of a universal set. Prove that $A=B$ iff $A\Delta B=\phi$, where $A\Delta B=(A-B)\cup(B-A)$.

P.T.O.

(e) In a group of 6 people, prove that there are three mutual friends or three mutual strangers.

(f) Show that

$$p \to q \equiv \neg q \to \neg p.$$

(g) Let $S = \{a, b, c\}$. Consider the poset $P(S)$ with the partial order as $\subseteq$. Verify that $(P(S), \cup, \cap)$ is a distributive lattice.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

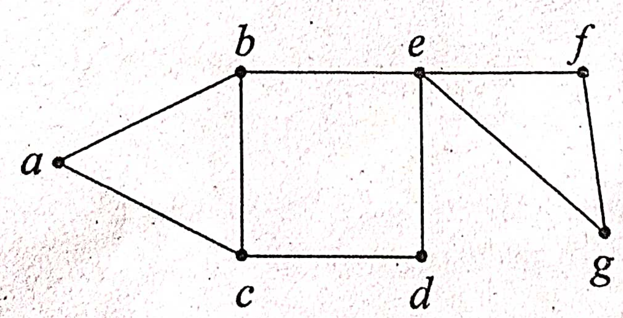
2. Let $A = \{1, 2, 3, 4\}$. Give example of a relation in A which is —

   (i) partial order relation,

   (ii) reflexive, transitive but not symmetric,

   (iii) transitive but not reflexive or symmetric.

2+1½+1½

3. 

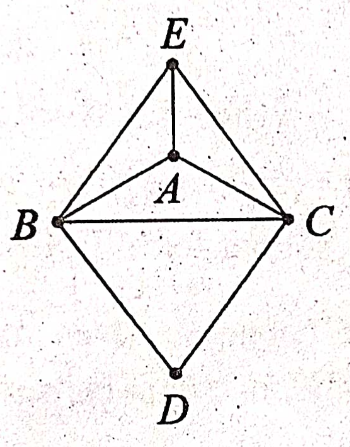
For the above graph, determine the following :

(a) a path from b to d,

(b) a closed walk from b to d that is not a circuit,

(c) a circuit from b to b that is not a cycle. 1+2+2

4. Consider the following graph.



(a) Is it Hamiltonian? Justify.

(b) Is there a Hamiltonian Path? Justify.

(c) Is it Eulerian? Justify.

(d) Is there an Eulerian trail? Justify. 1+2+1+1

5. Let p, q and r be primitive statements. Verify that each of the following is a tautology or not.

(i) $[p\vee(q\wedge r)]\vee\neg[p\vee(q\wedge r)]$

(ii) $[(p\vee q)\rightarrow r]\leftrightarrow[\neg r\rightarrow\neg(p\vee q)]$ 2+3

P.T.O.

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Find the *K*-map for the following expression :

$$x'y'zw + x'yzw' + xy'zw + xyzw'.$$ 4

(b) Express the Boolean function $f(x, y, z) = (x+y+z)(xy+xz)$; $x, y, z \in B$ in disjunction normal form (DNF). 5

7. (a) Identify the bound variables and the free variables in each of the following expressions.

(i) $\forall y \exists z [\cos(x+y) = \sin(z-x)]$

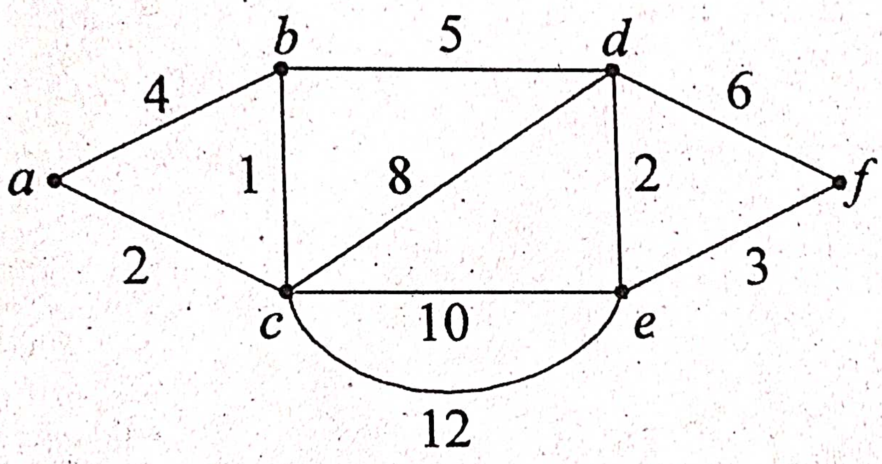
(ii) $\exists x \, \exists y [x^2 - y^2 = z]$

In both cases the universe comprises all real numbers. 2+2

(b) Find the solution to the recurrence relation, $a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \geq 0$, $a_0 = 3$, $a_1 = 7$. 5

8. (a) Find the maximum and minimum number of edges of a simple graph with 12 vertices and 4 components. 4

(b) Using Dijkstra's algorithm to find the length of the shortest path of the following graph from the vertex *a* to *f*: 5



UG/5th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH DC-12

(Numerical Methods and C Programming Language)

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

[Examinees are allowed to use Non-Programmable Calculator for Numerical Calculation.]

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) How many significant digits does the floating point number 0.03140×10^3 have?

(b) Prove that $(1+\Delta)(1-\nabla)=1$.

(c) State the conditions under which Newton-Raphson method fails to solve a polynomial equation.

(d) Evaluate $\int_0^1 x^2 dx$ by Trapezoidal Rule taking 5 sub-intervals and find the relative error in the result.

P.T.O.

(e) Write the error term of Simpson's 1/3 rule.

(f) What is the difference between *abs*() and *fabs*() functions?

(g) What are the derived data types in C-programming language?

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Using Newton-Raphson method, find the value of $\sqrt[5]{2}$, correct upto 5 significant figures. 5

3. Derive the total number of operations required in Gauss-Elimination method. 5

4. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's 3/8 rule. 5

5. Using Lagrange's interpolation formula, find the value of $f(x)$ for $x = 1$ from the following table : 5

x	-1	0	2	5
$f(x)$	9	5	3	15

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Derive Newton's Forward Interpolation formula. State when it can be used. 7

   (b) Find the value of $\left(\frac{\Delta}{E}\right)\sin(2x)$. 2

7. (a) Using 4th order Runge-Kutta method, find an approximate value of y for $x=0.2$ if $\frac{dy}{dx}=x+y^2$, given that $y=1$ when $x=0$ and $h=0.1$. 5

   (b) Define rate of convergence. Determine the rate of convergence for the Secant method. 4

8. (a) Write a program in C to find the value of $n\,!$. 4

   (b) What are the differences between 'while' and 'if' statements in C-programming? 3

   (c) Write a short note on control-string in C-programming. 2

UG/5th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH DC-11

(Advanced Analysis on $\mathbb{R}$ & $\mathbb{C}$)

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Is it possible to define a metric on any non-empty set? Justify your answer.

(b) If f be an analytic function on a region $G(\subset \mathbb{C})$ such that $Im f = 0$, then show that f is constant.

(c) Find the Laurent series expansion of the function $\frac{7z-2}{z(z-2)(z+1)}$ in the domain $|z|>2$.

(d) Describe explicitly, the ball $B(x_0, r)$ in the metric space $(\mathbb{R}, d)$, where $d(x, y)=|x-y|$.

P.T.O.

(e) Let $d(x, y) = \left|x^2 - y^2\right|$, $x, y \in \mathbb{R}$. Verify d is a metric on $\mathbb{R}$ or not.

(f) Examine the analyticity of the function $f(z) = |z-1|^2$ at $z = 1, 2$.

(g) Show that the subset $A = [0, 1)$ of the metric space (X, d) where $X = [0, 2)$ and 'd' is the usual metric is an open set.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Let $X = l_p (1 \le p < \infty)$, set of all pth summable sequences of real or complex numbers and let

$$d(x, y) = \left\{ \sum_{i=1}^{\infty} |x_i - y_i|^p \right\}^{\frac{1}{p}} \quad \text{where} \quad x = \{x_n\} \quad \text{and}$$

$y = \{y_n\} \in l_p$. Show that 'd' is a metric on $X = l_p$. 5

3. Prove that for a complete metric space (X, d), every nested sequence of non-empty closed sets $\{F_n\}$ with $diam(F_n) \to 0$ as $n \to \infty$, has a non-empty intersection containing precisely one point. 5

4. (a) Suppose $f(z) = u(x, y) + iv(x, y)$ and $\overline{f(z)}$

are both analytic throughout their domain D. Show that $f(z)$ must be constant throughout D. 3

(b) Show that the function $f(z)=|z|^2$ is nowhere differentiable except at the origin. 2

5. Show that the function $u(x, y)=\frac{y}{x^2+y^2}$ is harmonic in some domain and find σ a harmonic conjugate of $u(x, y)$. 5

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Let $X=\left\{1, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}, \cdots\right\}$ and 'd' is the usual metric defined on X. Let

$A=\left\{1, \frac{1}{3}, \frac{1}{5}, \cdots, \frac{1}{2n-1}, \cdots\right\}$ and

$B=\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \cdots, \frac{1}{2n}, \cdots\right\}$.

Find the distance between A and B. 3

(b) Evaluate $\int_C \frac{\cosh(\pi z)}{z(z^2+1)} dz$ using Cauchy's integral formula when $C:|z|=2$. 3

(c) Prove that every compact metric space is separable. 3

P.T.O.

7. (a) Consider any non-empty set X together with the metric

$$d(x, y) = \begin{cases} 1 & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$$

Examine whether (X, d) is complete or not. 3

(b) Let (X, d) be a metric space and A be a non-empty subset of X. Then show that (X, d) is connected if and only if every continuous mapping $f: A \to \{0, 1\}$ is a constant mapping. 3

(c) Find the value of the integral

$$\int_0^{1+i} \left(x - y + ix^2\right) dx$$

along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$. 3

8. (a) Let (X, d) and (Y, d') be two metric spaces. Show that a function $f: X \to Y$ is continuous if and only if for any $x \in X$ and for all sequence $\{x_n\}_{n \in \mathbb{N}}$ converges to x in (X, d), the sequence $\{f(x_n)\}_{n \in \mathbb{N}}$ converges to $f(x)$ in (Y, d'). 6

(b) Show that $\int_C (z - z_0)^n \, dz = \begin{cases} 2\pi i, & \text{if } n = -1 \\ 0 & \text{if } n \neq -1 \end{cases}$

where C is the circle with centre z_0 and radius $r > 0$ traversed in the anti-clockwise direction. 3

UG/5th Sem (H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MTMH DSE-2A/2B/2C

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

DSE 2A

(Differential Geometry)

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Calculate $g = \left| g_{ij} \right|$ when

$$ds^2 = (dx)^2 + 2\cos\phi dxdy + (dy)^2.$$

(b) Let $I \subset \mathbb{R}$ be an open interval and let x, $\tau : I \to \mathbb{R}$ be two differentiable functions such that $x(s) > 0$ for all $s \in I$. Does there exist a curve in $\mathbb{R}^3$ with x as its curvature and τ as its torsion?

P.T.O.

(c) For which condition a space curve (i.e., a curve in $\mathbb{R}^3$) reduces into a plane curve (i.e., a curve lying on a plane)?

(d) Does the first fundamental form of a surface in $\mathbb{R}^3$ depend on the parametrization of the surface?

(e) Mention true or false : Any surface in $\mathbb{R}^3$ has an empty interior when thought of as a subset of $\mathbb{R}^3$.

(f) Find the curvature of the space curve $r(t)=(a, bt, c/t)$ at $t=1$.

(g) Does the Weingarten map for a surface in $\mathbb{R}^3$ change sign when the orientation of the surface changes?

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Calculate the components of Riemann tensor for the metric

$$ds^2 = dx^2 + f(x, y)dy^2,$$

where f is a smooth function of x and y.

3. State and prove the Serret-Frenet formulae for a regular curve in $\mathbb{R}^3$.

4. Calculate the first and second fundamental form for the surface of revolution

$$\sigma(u, v) = (f(u)\cos v,\ f(u)\sin v,\ g(u)),$$

where f and g are continuously differentiable functions.

5. Let S be a regular surface in $\mathbb{R}^3$. Prove that S is orientable if and only if there exists a continuous function $N : S \to S^2$, where S^2 is the unit sphere, such that $N(p)$ is normal to S at p for each $p \in S$.

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Let $\gamma : I \to \mathbb{R}^3$ be a curve in $\mathbb{R}^3$ parametrized by arc length. Prove that γ is a straight line or a segment of a straight line in $\mathbb{R}^3$ if and only if its curvature vanishes everywhere. 4

(b) Consider as a surface S the hyperbolic paraboloid given by

$$S = \{(x, y, z) \in \mathbb{R}^3 : z = x^2 - y^2\}.$$

Study the second fundamental form of S at (0, 0, 0) and show that its Gauss curvature at (0, 0, 0) is negative. 5

P.T.O.

7. (a) Show that all contravariant transformations form a group under suitable composition. 5

(b) Prove that for any two surfaces S_1 and S_2 in $\mathbb{R}^3$, a local diffeomorphism $f : S_1 \to S_2$ is a local isometry if for any patch σ of S_1, the patches σ_1 and $f \circ \sigma_1$ of S_1 and S_2, respectively, have the same first fundamental form. 4

8. (a) Calculate the Frenet Apparatus $\{T, N, B, x, \tau\}$ of the curve γ in $\mathbb{R}^3$ given by

$$\gamma(t) = \left(t - \sin t \cos t,\ \sin^2 t,\ \cos t\right),\ t \in (0, \pi).$$ 5

(b) Prove that any tangent developable is locally isometric to a plane. 4

DSE 2B

(Fluid Mechanics)

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) If the velocity distribution is $u = iAx^2y + jBy^2zt + kCzt^2$, where A, B, C are constants, then find the acceleration and velocity components.

(b) Why is the centre of pressure is below the centre of gravity for an incline surface?

(c) Define steady and unsteady flow with examples.

(d) Determine the acceleration at the point (2, 1, 3) at $t = 0.5$ sec, if $u = yz + t$, $v = xz - t$ and $w = xy$.

(e) Show that there exist surfaces which cut streamline orthogonally if the velocity potential exist.

(f) Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 1.9.

(g) Show that free surface of a heavy homogeneous liquid at rest under gravity is horizontal.

P.T.O.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. In the two-dimensional motion of a liquid, if the current coordinates (x, y) are expressible in the forms of initial coordinate (a, b) and the time, then show that the motion is irrational if

$$\frac{\partial(\dot{x}, x)}{\partial(a, b)}+\frac{\partial(\dot{y}, y)}{\partial(a, b)}=0. \qquad 5$$

3. A mass of fluid is in motion so that the line of motion lie on the surface of co-axial cylinders. Show that the equation of continuity is

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho u)+\frac{\partial}{\partial z}(\rho v)=0$$

where u, v are the velocity perpendicular and parallel to z. 5

4. Prove that if the forces per unit mass at the point (x, y, z) parallel to the axes are $y(a-z)$, $x(a-z)$, xy; then the surface of equal pressure represents a hyperbolic paraboloid. 5

5. A semi-circular area is completely immersed in water with its plane vertical, so that the extremity A of its bounding diameter is in surface and the diameter makes

with the surface an angle α. Prove that if E be the C.P. and θ the angle between AE and the diameter,

$$\tan\theta = \frac{3\pi + 16\tan\alpha}{16 + 15\pi\tan\alpha}. \qquad 5$$

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) A semi-circular tube has its bounding diameter horizontal and contains equal volumes of n fluids of densities successively equal to $\rho, 2\rho, 3\rho ...$; arranged in this order. If each fluid subtend an angle 2α at the centre and the tube just holds them all, show that $\tan n\alpha = (2n+1)\tan\alpha$. 5

   (b) Given $u = \frac{-c^2 y}{r^2}$, $v = \frac{c^2 x}{r^2}$, $w = 0$, where r denotes distance from z-axis. Find the surfaces which are orthogonal to streamlines, the liquid being homogeneous. 4

7. (a) Prove that the acceleration of a fluid particle at P is given by

$$\vec{f} = \frac{\partial \vec{u}}{\partial t} + \text{grad}\left(\frac{1}{2}\vec{u}^2\right) - \vec{u} \times \text{curl}\,\vec{u}. \qquad 4$$

P.T.O.

(b) Determine the constants l, m, n in order that the velocity $\vec{q}=\left\{(x+lr)\hat{i}+(y+mr)\hat{j}+(z+nr)\hat{k}\right\}/\left\{r(n+r)\right\}$, where $r=\left(x^2+y^2+z^2\right)^{\frac{1}{2}}$ may satisfy the equation of continuity of a liquid. 5

8. (a) If a mass of liquid is in equilibrium under a given force $\vec{F}$ whose components per unit mass at the point (x, y, z) parallel to the coordinate axes are (X, Y, Z), then prove that $\vec{F}\cdot\left(\vec{\nabla}\times\vec{F}\right)=0$. 4

(b) Find the condition of equilibrium for homogeneous and heterogeneous fluid. 5

$\rho\left(X\,dx + Y\,dy + Z\,dz\right) = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz$

$Y = x(a-z) = ax - zx$

$\therefore \frac{\partial Y}{\partial z} = 0 - x \qquad \rho X = \frac{\partial p}{\partial x}$

$\frac{\partial Z}{\partial x} = a \qquad$ y [illegible]] $\qquad \frac{\partial p}{\partial y} = \rho Y$

DSE 2C

(Portfolio Optimization)

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) What do you mean by risk-free asset?

(b) By which measure of dispersion we calculate portfolio risk?

(c) Define Sharpe Ratio.

(d) What is market index?

(e) When an Investor agree to invest in high risk investments?

(f) How expected return (ER) is calculated?

(g) Write down mathematical formula to calculate the returns from Mutual Funds.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. An investor combine securities M & N and resulting portfolio is risk free. The variance of N is nine times larger than the variance of M. The expected return of

P.T.O.

M & N are 15% & 35% respectively. Find the expected return of the portfolio.

3. Briefly explain capital market line (CML).

4. What is the standard deviation (SD) of a random variable q with the following probability distribution?

Value of q	Probability
0	0.25
1	0.25
2	0.50

5. You have a portfolio with beta of 0.84. What will be the new portfolio beta if you keep 85% of your money in the old portfolio and in a stock with a beta of 1.93.

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Write down the objective of Investment.

(b) Suppose security A expected to produce annual return of 18%, 14% and 8% per annum during boom, normal and slump period respectively. Security B expected to earn annual return of 12%, 8%, 8% per annum during boom, normal and slump period. The probability of the economy being in the state of boom, normal and slump in the next period is 30%, 50% and 20% respectively. Explain the diversification with this situation. 2+7

7. Consider the following information about the return on classic mutual fund, the market return and the treasury bill (T bill) returns.

Year	Return of Classic mutual fund	Market return	T bill return
1994	17.1	10.8	5.4
1995	–14.6	–8.5	6.7
1996	1.7	3.5	6.5
1997	8.0	14.1	4.3
1998	11.5	18.7	4.7
1999	–5.8	–14.5	7.0
2000	–15.6	–26.0	7.9
2001	38.5	36.9	5.8
2002	33.2	23.6	5.0
2003	–7.0	–7.2	5.3
2004	2.9	7.4	6.2
2005	27.4	18.2	10.0
2006	23.0	31.5	11.4
2007	–0.6	–4.9	14.1
2008	21.4	20.4	10.7

Calculate SHARPE RATIO. 9

8. Briefly explain Capital Asset Pricing Model (CAPM). 9

2023

MATHEMATICS (Honours)

Paper Code : MTMH DSE-1A/1B/1C

Full Marks : 32 Time : Two Hours

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Notations and symbols have their usual meanings.

DSE 1A

(Advanced Algebra)

Group - A

(4 Marks)

1. Answer any *four* questions : $1\times4=4$

   (a) Prove that $Inn(S_3) \simeq S_3$.

   (b) Show that $\langle x^2+1 \rangle$ is not a prime ideal of $\mathbb{Z}_2[x]$.

   (c) Define Commutator subgroup of a group G.

   (d) Prove that for any group G, $|G/Z(G)| \neq 91$.

P.T.O.

(e) If H_1, H_2 be normal subgroups of G and G is an internal direct product of H_1 and H_2, then show that

$$G = H_1H_2 \text{ and } H_1 \cap H_2 = \{e\}.$$

(f) Prove that every group of order 77 is cyclic.

(g) Show that the polynomial $2x^5 + 15x^3 + 10x + 5$ is irreducible over $\mathbb{Z}$.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. (a) Let G be a group. Then prove that $Inn(G)$ is a normal subgroup of $Aut(G)$.

(b) Let G be an infinite cyclic group. Then prove that $Aut(G) \simeq \mathbb{Z}_2$. 2+3

3. (a) Let G be a non-commutative group of order p^3, where p is a prime. Prove that $|Z(G)| = p$.

(b) Find all Sylow 3-subgroups of S_4. 2+3

4. Show that no group of order 56 is a simple group. 5

5. Let for a group G, $\circ(G) = pq$, where p, q are distinct primes, $p < q$, $p \nmid (q-1)$. Show that G is cyclic. 5

Group - C

(18 Marks)

Answer any *two* questions : $9\times2=18$

6. (a) Let for a group G, $\circ(G)=p^n$, p being prime. If H is a subgroup of G such that $\circ(H)=p^{n-1}$, show that H is normal in G. 4

   (b) Let G be a group of order 100. If G has a unique Sylow 2-subgroup, then prove that G is a commutative group. 3

   (c) Let G be a group such that $Aut(G)=\{I_G\}$ where I_G denotes the identity mapping on G. Prove that G is a commutative group and $a^2=e$ for all $a\in G$. 2

7. (a) Prove that every group of order p^2, where p is a prime, is commutative. 3

   (b) Determine whether $4=1+1+2$ can be the class equation of a group. Justify your answer. 1

   (c) Prove that every Euclidean domain is a PID. Show that $\mathbb{R}[x]$ is a PID but $\mathbb{Z}[x]$ is not a PID. 3+1

   (d) Let G be a simple group of order 168. Show that G has eight Sylow 7-subgroups. 1

P.T.O.

8. (a) Find all irreducible polynomials of degree 2 in $\mathbb{Z}_2[x]$. 3

(b) Examine whether 3 is a prime element in the integral domain $\mathbb{Z}\left[\sqrt{-3}\right]$ or not. 4

(c) If R is an integral domain, then prove that $R[x]$ is also an integral domain. 2

DSE 1B

(Number Theory)

Group - A

(4 Marks)

1. Answer any *four* questions : $1\times4=4$

(a) Determine the smallest positive integer having only 10 positive divisors.

(b) Show that $x^2+y^2=43$ cannot have any integral solution. Explain.

(c) Find the order of $\overline{5}$ in $\mathbb{Z}_{17}$.

(d) For $n=3000$, find $d(n)$ and $\sigma(n)$.

(e) Determine $\sum_{j=1}^{n}\mu(j!)$, where μ is the Möbius function.

(f) Solve $x^2\equiv 5(\text{mod}\,29)$.

(g) Find the remainder when 2^{50} is divided by 7.

P.T.O.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Define Euler ϕ-function. Let n be the positive integers of the form $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_r^{\alpha_r}$, where $p_1, p_2, \ldots, p_r$ are distinct primes and each α_i are greater than or equal to 1. Prove that

$$\phi(n) = n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots\cdots\left(1-\frac{1}{p_r}\right).$$ 5

3. If p is an odd prime and p does not divide a, then $x^2 \equiv a \pmod{p}$ has a solution or no solution depending on whether $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$ or $a^{\frac{p-1}{2}} \equiv -1 \pmod{p}$, respectively. 5

4. Solve the system of linear congruences

$x \equiv 3(\text{mod}\,11)$, $x \equiv 5(\text{mod}\,19)$, $x \equiv 10(\text{mod}\,29)$. 5

5. Decipher the message

"TZSVIW JQBVMIJ ALMVOOVI"

which was produced using the cipher $C = 3p + 7(\text{mod}\,26)$. 5

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) If p be a prime number, prove that $(p-1)!+1 \equiv 0 \pmod{p}$. 4

   (b) If p be a prime and a is prime to p, prove that

   $$a^{p^2-p} \equiv 1 \left(\bmod p^2\right).$$ 2

   (c) If p is an odd integer, then by using Wilson's theorem show that

   $$1^2.3^2.5^2.\cdots(p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod{p}.$$ 3

7. (a) For any positive integer n, show that $n = \sum_{d|n} \phi(d)$. 3

   (b) If F and f are two number theoretic functions related by the formula $F(n) = \sum_{d|n} f(d)$, then show that $f(n) = \sum_{d|n} \mu(d) \cdot F\left(\frac{n}{d}\right)$, where μ is the Möbius function. Hence show that

   $$\phi(n) = \sum_{d|n} \mu(d) \cdot \frac{n}{d}.$$ 3+1=4

P.T.O.

(c) If (x, y, z) is primitive Pythagorean triplet, prove that both x and y cannot be even integers. 2

8. (a) Show that every prime p has $\phi(p-1)$ primitive roots. 3

(b) Encrypt the message "NO WAY" by RSA system. 3

(c) Find the general solution in integers and the least positive integral solutions of the equation $35x - 13y = 10$. 3

DSE 1C

(Bio-Mathematics)

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

   (a) What is an endemic?

   (b) Define Allee Effect.

   (c) Find the nature and stability of the fixed point of $\dot{x} = x - by,\ \dot{y} = bx + y$, for different values of b.

   (d) Define Horizontal transmission.

   (e) The system has the characteristic equation
   $$\lambda^3 + 4K\lambda^2 + (5+K)\lambda + 10 = 0.$$
   Find the range of K for which the system is stable?

   (f) State the Routh-Hurwitz criteria of order 3.

   (g) Write down a two-species model with diffusion.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Write down the assumptions, draw the schematic diagram and formulate bacterial growth model in a chemostat. 5

P.T.O.

3. What is insect outbreak? Give an example. Write down the insect outbreak model. 5

4. Find the fixed point and investigate their stability for the following logistic map.

$$x_{n+1} = r x_n (1 - x_n), \ r > 0. \qquad 2+3$$

5. Consider the growth model

$$\frac{dN}{dt} = rN\left(\frac{N}{A} - 1\right)\left(1 - \frac{N}{K}\right),$$

where r, A, K are positive parameters and $A < K$. Determine all the equilibrium points and check their stability. 3+2

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Consider the difference equation

$$x_{n+1} = 0.5x_n \text{ with } x_0 = 1024.$$

solve the dynamical system and compute x_{10}.

(b) Consider the Nicholson-Bailey host-parasitoid model as

$$H_{t+1} = K\, H_t e^{-aP_t}$$

$$P_{t+1} = c\, H_t \left(1 - e^{-aP_t}\right)$$

where H_t and P_t be the host and parasitoid population size at time t. Here a is the searching efficiency of the parasitoid and c be the number of viable eggs which parasitoid lays on a single host.

Find the fixed points and investigate the stability property of them. 3+6

7. Consider the following system :

$$\frac{dx}{dt} = x(1-x-y)$$

$$\frac{dy}{dt} = \beta(x-\alpha)y.$$

(a) Show that the system contains no periodic solution within the first quadrant. Here α and β are positive constants.

(b) Also find the equilibrium points of the system and discuss their stability. 5+4

8. (a) Write down a single-species harvesting model.

(b) Find the maximum sustainable yield and the optimal harvesting rate.

(c) Explore transcritical bifurcation if exist. 2+4+3