UG/3rd Sem (H/G)/22/(CBCS)

2022

MATHEMATICS (Honours/General)

Paper Code : MTMG DC-3/GE-3

[Vector Algebra & Multivariate Calculus]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(4 marks)

1. Answer any ***four*** questions : 1×4=4

(a) If $|\vec{\alpha}|=3$ and $|\vec{\beta}|=4$ then find the values of the scalar c such that the vectors $\vec{\alpha}+c\vec{\beta}$ and $\vec{\alpha}-c\vec{\beta}$ will be orthogonal to one another.

(b) Define curl of a vector point function.

(c) State Youngs theorem for functions of two variables.

(d) Define homogeneous function of degree n in three variables.

P.T.O.

(e) Evaluate $\int_0^1 \int_0^2 x^3 y \, dx \, dy$.

(f) Let $f(x,y,z) = \frac{x^2}{y} + \frac{y^2}{z} + \frac{z^2}{x}$, $xyz \neq 0$, without computing fx, fy, fz find $xf_x + yf_y + zf_z$.

(g) If $u = x^2 y + y^2 z + z^2 x$, show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x+y+z)^2.$$

Group - B

(10 marks)

Answer any *two* questions : 5×2=10

2. Prove that

$$(\vec{b} \times \vec{c}) \cdot (\vec{a} \times \vec{d}) + (\vec{c} \times \vec{a}) \cdot (\vec{b} \times \vec{d}) + (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 0.$$

3. Find div $\vec{F}$ and curl $\vec{F}$, where

$$\vec{F} = \text{grad}\left(x^3 + y^3 + z^3 - 3xyz\right).$$

4. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Find the value of $\vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} + \vec{a} \cdot \vec{b}$.

5. If $f(x,y)=\dfrac{x^2y^2}{x^2+y^2}$, $x^2+y^2\neq 0$

$$= 0, \qquad x^2+y^2=0$$

then show that $f_{xy}(0,0) = f_{yx}(0,0)$.

Group - C

(18 marks)

Answer any *two* questions : 9×2=18

6. (a) Evaluate $\iint xy(x+y)\,dx\,dy$ over the region bounded by $y=x^2$ and $y=x$. 5

(b) Find a unit vector in the plane of the vectors $\vec{i}+2\vec{j}-\vec{k}$ and $\vec{i}+\vec{j}-2\vec{k}$, which is perpendicular to the vector $2\vec{i}-\vec{j}+\vec{k}$. 4

7. (a) Examine the existence of maxima or minima of the function $f(x,y)=xy$ subject to the condition $5x+y=13$. 4

(b) If $u=\sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$ then prove that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u.$$ 5

P.T.O.

8. (a) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$, prove that grad $f(r) \times \vec{r} = \vec{0}$. 5

(b) Show that the mapping $T : R^3 \to R^3$ defined by

$T(x_1, x_2, x_3) =$

$(x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3)$,

$(x_1, x_2, x_3) \in R^3$ is a linear mapping. 4
