UG/1st Sem (H)/22/(CBCS)

2022

MATHEMATICS (Honours)

Paper Code : MTMH DC-1

(Calculus & Geometry)

Full Marks : 32 | Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Show that $f(x)=\begin{cases}\sin x\sin\left(\dfrac{1}{\sin x}\right), & x\neq 0\\ 0, & x=0\end{cases}$

is continuous at $x = 0$.

(b) If $y=\dfrac{x^3}{x^2-1}$, then find $(y_n)_0$, where $x > 1$.

(c) Find the oblique asymptote of the curve $y = xe^{\frac{1}{x}}$

(d) Show that $6x^2-5xy-6y^2+14x+5y+4=0$ represents a pair of straight lines perpendicular to each other.

P.T.O.

(e) Find the eccentricity of the conic $r = \dfrac{9}{2+\cos\theta}$

(f) Find the radius of curvature at (0, 0) of the curve

$$y - x = x^2 + 2xy + y^2$$

(g) Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x\, dx$.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. If $y = \left(x + \sqrt{x^2 - 1}\right)^m$, prove that

$$\left(x^2 - 1\right) y_{n+2} + (2n+1) x y_{n+1} + \left(n^2 - m^2\right) y_n = 0.$$ 5

3. Show that the equation of the cone whose vertex is the origin and base is the curve $f(x, y) = 0,\ z = k$ is given by $f\left(\dfrac{kx}{z}, \dfrac{ky}{z}\right) = 0.$ 5

4. Tangents are drawn from (h, k) to the circle $x^2 + y^2 = a^2$. Find the area of the triangle formed by them and the straight line joining their points of contact. 5

5. If $f : [a, b] \to \mathbb{R}$ be continuous on $[a, b]$ and $f(a).f(b) < 0$, then show that there exists at least one point c in (a, b) such that $f(c) = 0$. 5

3

Group - C

(18 Marks)

Answer any *two* questions. 9×2=18

6. (a) Let $f:[0,1]\rightarrow\mathbb{R}$ is continuous on [0, 1] and f assumes only rational values. If $f\left(\frac{1}{2}\right)=\frac{1}{2}$, prove that $f(x)=\frac{1}{2}$ for all $x\in[0,1]$. 5

(b) If $I_n=\int_0^{\frac{\pi}{2}} x^n \sin x\, dx$, n being a positive integer > 1. Prove that $I_n+n(n-1)I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$.

4

7. (a) Obtain the equations to the generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{a^2}=1$ that pass through the point $(a\cos\alpha, b\sin\alpha, 0)$. 5

(b) Given that the astroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=c^{\frac{2}{3}}$ is the envelope of the lines $\frac{x}{a}+\frac{y}{b}=1$, prove that the parameters are connected by the relation $a^2+b^2=c^2$. 4

P.T.O.

8. (a) Reduce the equation $x^2 + y^2 + z^2 - 2xy - 2yz + 2zx + x - 4y + z + 1 = 0$ to its canonical form and determine the type of the quadric represented by it. 5+1

(b) Find the area of the loop of the curve

$$x(x^2 + y^2) = a(x^2 - y^2)$$ 3

5

UG/1st Sem (H)/22/(CBCS)

2022

MATHEMATICS (Honours)

Paper Code : MTMH DC-2

(Algebra)

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(4 Marks)

1. Answer any *four* questions : $1\times4=4$

(a) If $z\neq0$ is a complex number such that $\arg(z)=\frac{\pi}{4}$, then show that $\operatorname{Re}(z^2)=0$.

(b) Find the sum of 99^{th} powers of the roots of the equation $x^7-1=0$.

(c) If A and B are real orthogonal matrices of same order and if $C=BAB^{-1}$ and λ be any scalar, then prove that $\det.(c+\lambda I)=\det.(A+\lambda I)$.

(d) Prove that, $(x^2y+y^2z+z^2x)(xy^2+yz^2+zx^2)\geq 9x^2y^2z^2$, where x, y, z are positive.

P.T.O.

(e) Prove that the product of three consecutive positive integers is always divisible by 6.

(f) If $A = \{x : -1 \le x \le 1\}$ and $f : A \to A$ is a function, then show that $f(x) = x|x|$ is a bijection.

(g) If p be a prime and $p|ab$, then show that either $p|a$ or $p|b$.

Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. If α, β, γ be the roots of the equation $x^3 + px^2 + 2x + r = 0$, then find the equation whose roots are $\beta^2 + \gamma^2 - \alpha^2, \gamma^2 + \alpha^2 - \beta^2, \alpha^2 + \beta^2 - \gamma^2$.

3. Use De Moiver's theorem to prove that $\cos^n \theta = 2^{1-n}$ $\left[\cos n\theta + {}^nc_1 \cos(n-2)\theta + {}^nc_2 \cos(n-4)\theta + ...\right]$, where n is a positive integer and θ is real.

4. Find the minimum value of $3x + 2y$ where x, y are positive real numbers satisfying the condition $x^2y^3 = 48$.

5. Express the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{bmatrix}$ as a product of elementary matrices and hence find A^{-1}.

7

Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ 5

(b) Define partition of a set $S(\neq \phi)$. If R be an equivalence relation on S, then show that R determines a partition of S. 4

7. (a) Solve the equation

$6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$. 4

(b) If a, b, c are positive integers such that $gcd(a, b) = 1 = gcd(a, c)$, then prove that $gcd(a, bc) = 1$. 2

(c) If $p > 2$ be a prime, then prove that

$1^p + 2^p + 3^p + \ldots + (p-1)^p \equiv 0 \pmod{p}$ 3

8. (a) Show that the ratio of the principal values of $(1+i)^{1-i}$ and $(1-i)^{1+i}$ is

$\sin(\log 2) + i\cos(\log 2)$ 5

P.T.O.

(b) Solve the system of linear equations by matrix method : 4

$$
\begin{aligned}
x + 2y + z &= 1 \\
3x + y + 2z &= 3 \\
x + 7y + 2z &= 1
\end{aligned}
$$
