Gour Mahavidyalaya
Department of Mathematics

UG/5th Sem (H)/22/(CBCS)

2022

# MATHEMATICS (Honours)

## Paper Code : MTMH DC-11

## (Advanced Analysis on $\mathbb{R}$ & $\mathbb{C}$)

Full Marks : 32 Time : Two Hours

*The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.*

Notations and symbols have their usual meaning.

### Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Verify whether $f(z)=\overline{z}$ is analytic or not.

(b) Find the radius of convergence of the following power series $\sum_{n=1}^{\infty}\frac{z^{2n}}{2^n}$.

(c) Find the period of the function $f(z)=e^{iz}$.

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(d) Prove that the function $f(z)=\frac{1}{z}$ is continuous for all $z$ such that $|z|>0$.

(e) Give examples of a $G_\delta$ set and $F_\sigma$ set.

(f) Let $d(x, y)=|\sin(x-y)|$, $x, y \in \mathbb{R}$. Verify $d$ is a metric on $\mathbb{R}$ or not.

(g) Prove that $\mathbb{Q}$ with usual metric on $\mathbb{R}$ is not complete.

**Group - B**

(10 Marks)

Answer any *two* questions : 5×2=10

2. Consider the function $f$ defined by

$$f(z)=\begin{cases} 0, & \text{when } z=0 \\ \dfrac{x^3-y^3}{x^2+y^2}+i\dfrac{x^3+y^3}{x^2+y^2}, & z \neq 0. \end{cases}$$

Show that the function satisfies C-R equation but $f$ is not analytic at $z=0$. 5

3. If $f(z)$ is an analytic function of $z$, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|\operatorname{Re} f(z)|^2=2|f'(z)|^2.$$ 5

4. Show that the metric space $(C[a, b], d)$ is complete, where $C[a, b]$ is the collection of all real valued continuous functions defined on $[a, b]$ and the metric $d$ on $C[a, b]$ is defined as $d(f, g) = \sup\{|f(x) - g(x)| : a \le x \le b\}$, $f, g \in C[a, b]$. 5

5. Let $(X, d)$ be a metric space and a set $A$ is connected in $(X, d)$. If $A \subset G_1 \cup G_2$, where $G_1$ and $G_2$ are separated sets in $(X, d)$, then show that either $A \subset G_1$ or $A \subset G_2$. 5

## Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Show that the space $l_p$ with $1 \le p < \infty$ is separable. Give an example of a metric space which is not separable. 5

(b) Use Cauchy integral formula to evaluate the integral :

$$\int_{|z|=5} \frac{z+5}{z^2 - 3z - 4} dz.$$ 4

7. (a) Find the Laurent Series expansion of the function

$$f(z) = \frac{z}{z^2 - 4z + 3}, \text{ where } |z| > 3.$$ 4

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(b) Let $(X, d)$ be a metric space and $d' : X \times X \to \mathbb{R}$ defined by

$$d'(x,y) = \frac{d(x,y)}{1+d(x,y)},\ x, y \in X.$$

Show that $d'$ is a bounded metric on $X$. 5

8. (a) Let $u - v = e^x(\cos x - \sin y)$ and $f(z) = u + iv$ is an analytic function $z = x + iy$. Find $f(z)$ in terms of $z$. 4

(b) Prove that continuous image of a compact metric space is compact. 5

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# MATHEMATICS (Honours)

## Paper Code : MTMH DC-12

## (Numerical Methods and C Programming Language)

Full Marks : 32 Time : Two Hours

*The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.*

Notations and symbols have their usual meanings.

### Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Find the percentage error in $f(x)$ for $f(x)=2x^3-4x$ at $x=1$, when the error in $x$ is 0.04.

(b) Show that : $\Delta+\nabla=\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}$.

(c) Distinguish between the direct and iterative method for solving a system of linear equations.

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(d) Find the polynomial of degree less than or equal to 2 such that $f(0) = 1, f(1) = 3, f(3) = 151$.

(e) State sufficient condition for the convergence of fixed point iteration method.

(f) What is the difference between do-while and while statement?

(g) Define degree of precision of a quadrature formula. What is the degree of precision of Simpson's 1/3 rule?

## Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Deduce Newton's backward interpolation formula without error term. 5

3. Briefly describe the method of iteration to solve an equation $x = \phi(x)$. Also find the condition of convergence. 2+3

4. Describe the Gauss-Seidel method to solve a system of linear equation. Mention the condition of convergence of the method. 5

5. Write a programme in C to solve an equation $f(x) = 0$ using Newton Raphson method. 5

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## Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Solve the system of linear equations

$$2x_1 - 2x_2 + x_3 = 2$$
$$5x_1 + x_2 - 3x_3 = 0$$
$$3x_1 + 4x_2 + x_3 = 9$$

by matrix factorization method. 5

(b) Write a short note on 'if' statement and 'if-else' statement. 4

7. (a) Establish Lagrange's polynomial interpolation formula with remainder term. 5

(b) Write a program in C to find the sum

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$ 4

8. (a) Obtain Simpson's $\frac{1}{3}$ rule from Newton Cote's numerical integration formula. 5

(b) Evaluate $y(1.0)$ from the differential equation $\frac{dy}{dx} = y + x^2$ with $y(0) = 1$, taking $h = 0.5$ by 4-step Runge-Kutta method. 4

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# MATHEMATICS (Honours)

**Paper Code : MTMH DSE-1A/1B/1C**

Full Marks : 32 Time : Two Hours

*The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.*

Notations and symbols have their usual meanings.

## Paper : DSE 1A

## (Advanced Algebra)

### Group - A

### (4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Show that the given map is an automorphism

$G$ : Cyclic group of order 6

$f: G \to G$ s.t. $f(n) = n^5$.

(b) Show that the direct product $\mathbb{Z} \times \mathbb{Z}$ is not a cyclic group.

(c) Find the order of $(\overline{2}, \overline{1})$ of $z_3 \times z_5$.

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(d) Let $p$ be a prime and $n \geq m$ be nonnegative integers where

$$n = a_0 + a_1 p + a_2 p^2 + \cdots + a_k p^k$$
$$m = b_0 + b_1 p + b_2 p^2 + \cdots + b_k p^k$$
$$0 \leq a_i,\ b_i \leq p-1$$
$$\binom{a_0}{b_0}\binom{a_1}{b_1}\binom{a_2}{b_2}\cdots\binom{a_k}{b_k} \bmod (p) = ?$$

(e) Give example of a non abelian group of order 6.

(f) Define stabilizer of 'a' in a group.

(g) Give an example of a PID which is not a Euclidean domain.

**Group - B**

(10 Marks)

Answer any *two* questions : 5×2=10

2. Show that the set $I(G)$ of all inner automorphism of $G$ is a subgroup of set $G$.

3. Let a group $G$ acts on a set $X$. Then show that different orbits forms a partition of $X$.

4. Show that every non-cyclic group of order 21 contains only 14 elements of order 3.

5. Prove that a polynomial of degree $n$ over a field has at most $n$ zeros, counting multiplicity.

## Group - C

(18 Marks)

Answer any *two* questions : $9\times2=18$

6. (a) Let $G$ be a finite group. Let $H$ be normal in $G$. If $p$ be a prime dividing $\circ(G)$ such that $([G:H], p)=1$. Show that $H$ contains every Sylow $p$ subgroups of $G$. 3

   (b) Show that the polynomial $f(x)=2x^2+4$ is irreducible over $\mathbb{Q}$ but reducible over $\mathbb{Z}$. 3

   (c) Let $G$ be a group and $Z$, the centre of $G$. Then prove that the set of all inner automorphism is isomorphic to the quotient group $G/Z$. 3

7. (a) Find all the conjugate classes in $S_4$ and verify the class equation. 5

   (b) Let $R$ be a commutative ring with identity such that $R[n]$ is a principal ideal domain, then prove that $R$ is a field. 4

8. (a) Show that $\mathbb{Z}_2 \oplus \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_6$. $\oplus$ denotes the external direct product. 4

   (b) (i) Show that in an integral domain, every prime is an irreducible. 2

   (ii) If $F$ be a field, then show that $F[x]$ be a unique factorization domain. 3

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## Paper : DSE 1B

## (Number Theory)

### Group - A

### (4 Marks)

1. Answer any *four* questions : $1\times4=4$

   (a) Find the number of positive divisors of 2700.

   (b) State Chinese Remainder theorem.

   (c) Find the number of integers less than $n$ and prime to $n$, where $n=16$.

   (d) Show that 5 is a primitive root of 18.

   (e) Find the order of 5 modulo 19.

   (f) If $n$ is an odd integer, prove that $n^2 = 1 \bmod(8)$.

   (g) When is a number-theoretic function called multiplicative?

### Group - B

### (10 Marks)

Answer any *two* questions : $5\times2=10$

2. Prove that the Möbius $\mu$-function is a multiplicative number theoretic function. 5

3. Let $n>1$ be the positive integer of the form

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots, p_r^{\alpha_r}$$

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where $p_1, p_2, ..., p_r$ are distinct primes and each $\alpha_i$ is an integer greater than 1. Prove that $\tau(n) = \prod_{i=1}^{n}(\alpha_i + 2)$. 5

4. Decrypt the ciphertext

1030 1511 0744 1237 1719

that was encrypted using the RSA algorithm with key (2623, 869). 5

5. If $F_n = 2^{2^n} + 1$, $n > 1$ is a prime, then show that 2 is not a primitive roots of $F_n$. 5

## Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) If $(2n+1)$ is prime, prove that $(n!)^2 \equiv (-1)^{n+1} \pmod{2n+1}$. 4

(b) Solve the system of linear congruences

$$x \equiv 1 \pmod 3$$
$$x \equiv 2 \pmod 5$$
$$x \equiv 3 \pmod 7.$$ 5

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7. (a) Prove that the Euler's phi-function $\phi$ is a multiplicative function. 5

(b) Encrypt the message GOOD CHOICE using an exponential cipher with modulus $p = 2906$ and exponent $k = 7$. 4

8. (a) If $x, y, z$ is a primitive pythagorean triple, then show that one of the integers $x$ or $y$ is even while the other is odd. 4

(b) Determine all solutions in the integers of the linear Diophantine equation

$$221x + 35y = 11.$$ 5

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## Paper : DSE 1C

## (Bio Mathematics)

### Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Write down the assumptions of Malthusian population model.

(b) Describe Nicholson-Bailey model.

(c) Let $x^*$ be a fixed point of the difference equation $x_{n+1} = rf(x_n)$ and $x(t_0) = x_0$, when $x^*$ is said to be stable and attracting.

(d) What is holling functional response?

(e) Find the nature and stability of the fixed points of $x = -ax + y,\ y = -x - ay$ for different values of $a$.

(f) Define the carrying capacity.

(g) What is an pandemic?

### Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Consider the system of equations

$$\frac{dx}{dt} = 1 - (a+1)x + bx^2 y$$

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$$\frac{dy}{dt} = ax - bx^2 y$$

where $a$ and $b$ are positive parameters. Then linearize the system about its critical points. 5

3. Describe a logistic population model with constant harvesting rate and also find the condition under which population will be extinct. Find the equilibrium points. 2+3

4. Find the non-negative equilibrium point of a population model governed by the equation $x_{n+1} = \frac{2x_n^2}{x_n^2 + 3}$ and check their stability. 5

5. Determine the stability of the equilibrium point of the chemostat model. 5

## Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. Describe a predator-prey model with necessary assumptions. Find solution under different cases when prey population exists only or predator population exists only or both population exists. 2+2+2+3

7. With suitable assumptions write down the Lotka-Volterra prey-predator model and discuss the dynamical behaviour of the system around the equilibrium point. 9

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8\. (a) Find the equilibrium points and stability of the model $x_{n+1} = r(1 - x_n)x_n$ and also find the value of $r$ for which bifurcation occurs.

(b) What is the Allee effect in a population model and also its general characteristics. (2+2+2)+(2+1)

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# MATHEMATICS (Honours)

## Paper Code : MTMH DSE-2A/2B/2C

Full Marks : 32 Time : Two Hours

*The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.*

Notations and symbols have their usual meanings.

## Paper : DSE 2A

## (Differential Geometry)

### Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) What is geodesic in a surface?

(b) What is osculating circle?

(c) Find the curvature of the cubic curve given by $\bar{r} = (u, u^2, u^3)$.

(d) Show that "$\delta^i_j$" is a mixed tensor of order two.

(e) Show that for a plane curve the torsion $\tau = 0$.

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(f) In $V_4$ with line element

$$ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$$

show that the vector $\left(\sqrt{2}, 0, 0, \frac{\sqrt{3}}{c}\right)$ is a unit vector.

(g) Write down the Serret-Frenet formula for a space curve.

## Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Let us consider the three dimensional Euclidean space $E_3$. Find out the metric tensor in cylindrical polar co-ordinates. 5

3. If $A_i$ is a covariant vector, prove that $\left(\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}\right)$ is a covariant tensor of rank 2. 5

4. Prove that the surface $xy = (z-c)^2$ is developable. 5

5. Consider the curve $u = u(t)$, $v = v(t)$. Find the direction co-efficients of the tangent to the curve. 5

## Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) What is line of curvature on a surface? 1

(b) Write down the differential equation of lines of curvature and show that the equation can be put in the form

$$[N, dN, d\bar{r}] = 0,$$

where N denotes unit normal to the surface. 4

(c) Find the differential equation of line of curvature of the helicoid

$$x = u\cos v,\ y = u\sin v,\ z = f(u) + cv.$$ 4

7. (a) State the Clairaut's Theorem on geodesic. 1

(b) Show that the curves $u + v =$ costant, are geodesic on a surface with metric

$$(1+u^2)du^2 - 2uv\,du dv + (1+v^2)dv^2.$$ 5

(c) Prove that straight lines on a surface are the only asymptotic lines which are geodesics. 3

8. (a) Define Riemannian Metric. 2

(b) Show that the co-efficients $g_{ij}$ of Riemannian metric is a covariant tensor of rank 2. 2

(c) If $A^{ij}$ is a tensor of type (2, 0), prove that

$$A^{ij}{}_{,i} = \frac{1}{\sqrt{g}} \cdot \frac{\partial}{\partial x^i}\left(A^{ij}\sqrt{g}\right) + A^{ir}\begin{Bmatrix} j \\ ir \end{Bmatrix}.$$ 5

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## Paper : DSE 2B

## (Fluid Mechanics)

### Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

(a) Write down the difference between laminar and turbulent flows.

(b) A manometer is used to measure the pressure at a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.

(c) State the condition under which a meta centre will exist.

(d) What is the velocity profile for Poiseuille flow?

(e) Show that the external force is normal to the equi-pressure surface.

(f) State Reynolds transport theorem.

(g) Write the condition when a fluid is said to be incompressible.

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## Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. Assuming that the velocity components for a two dimensional flow system can be given in the Eulerian System by $u = A(x+y)+Ct$, $v = B(x+y)+Et$, find the displacement of a fluid particle in the Lagrangian System. 5

3. Show that for an incompressible steady flow with constant viscosity, the velocity components

$$u(y) = y\frac{U}{h}+\frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\cdot\frac{y}{h}\left(1-\frac{y}{h}\right), \quad v=0, \quad w=0,$$

satisfy the equation of motion, when the body force is neglected. $h$, $U$, $\frac{dp}{dx}$ are constant and $p = p(x)$. 5

4. If $q$ is the resultant velocity at any point of a fluid which is moving irrotationally (i.e., $q = -\nabla\phi$, $\phi$ is the velocity potential) in two dimensions. Prove that

$$\left(\frac{\partial q}{\partial x}\right)^2+\left(\frac{\partial q}{\partial y}\right)^2 = q\nabla^2 q\,.$$ 5

5. If $X$, $Y$, $Z$ be the external force components unit mass, then prove that

$$dp = \rho(Xdx+Ydy+Zdz)\,.$$ 5

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## Group - C

(18 Marks)

Answer any *two* questions : $9\times2=18$

6. (a) Show that the pressure at a point in a fluid in equilibrium is same in every direction. 4

(b) A triangular gate which has a base of 1.5 m and an altitude of 2 m lies in a vertical plane. The vertex of the gate is 1 m below the surface of a tank which contains oil of specific gravity 0.8. Find the force exerted by the oil on the gate and the position of the centre of pressure. 5

7. (a) If the velocity potential of a fluid is $\phi=\left(\frac{z}{r^3}\right)\tan^{-1}\left(\frac{y}{x}\right)$, where $r^2=x^2+y^2+z^2$, then show that the streamline lie on the surface $x^2+y^2+z^2=c\left(x^2+y^2\right)^{2/3}$, $c$ being an arbitrary constant. 4

(b) A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed exis, show that the equation of continuity is

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial\theta}(\rho w)=0,$$

where $w$ is the angular velocity of a particle whose azimuthal angle is $\theta$ in time $t$. 5

8. (a) Find the pressure at any point and the surface of equal pressure, when a mass of homogeneous liquid contained in a vessel revolves uniformly about a vertical axis. 5

(b) Fluid contained within a circular tube of radius $a$ in a vertical plane which can rotate about a vertical diameter. If the fluid subtend an angle $\theta$ at the centre, the least angular velocity to make the fluid divide into part is $\sqrt{\frac{g}{a}}\sec\left(\frac{\theta}{4}\right)$. 4

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## Paper : DSE 2C

## (Portfolio Optimization)

### Group - A

(4 Marks)

1. Answer any *four* questions : 1×4=4

   (a) What is capital market line?

   (b) State any source of business risk?

   (c) What is meant by investment?

   (d) State any two benefits of diversification.

   (e) What is Sharpe's risk index?

   (f) What do you mean by speculation?

   (g) What do you understand by financial assets?

### Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. What are the objective of industry analysis? 5

3. Briefly explain Markowitz Model of Portfolio Management. 5

4. What is investment decision making? Discuss various approaches to it. 5

5. What is systematic and unsystematic risk. 5

## Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Calculate the expected return and standard deviation of return for a stock having the following probability distribution of returns :

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Possible Returns (in %) : | 35 | 30 | 20 | 15 | 0 | –10 | –25 |
| Probability of Occurrence : | 0.15 | 0.20 | 0.25 | 0.15 | 0.10 | 0.10 | 0.05 |

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(b) What do you understand by financial assets? 2

7. Suppose your first job pays you $28,000 annually. What percentage should your cash reserve contain? How much life insurance should you carry if you are unmarried? If you are married then how much life insurance should you carry with two young children? 3+3+3

8. State assumptions made in "Capital Asset pricing model". What are the uses and limitations of this model? 9

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# MATHEMATICS (Honours)

## Paper Code : MTMH SEC-1

## (Discrete Mathematics)

Full Marks : 32 Time : Two Hours

*The figures in the margin indicate full marks.*
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### Group - A

(4 Marks)

1. Answer any *four* questions : $1\times4=4$

(a) Define path in a graph.

(b) Define Bipartite Graph.

(c) Prove that $P \vee P \Leftrightarrow P$ is a tautology.

(d) Obtain the disjunctive normal form of the statement $p \wedge (p \rightarrow q)$.

(e) Draw a simple connected graph in which the degree of each vertex is exactly 4.

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(f) Show that the relation $\geq$ is a partial order on the set of integers, $\mathbb{Z}$.

(g) Prove that in a Boolean algebra $(B, +, \cdot, ', 0, 1)$ $1 + a = 1 \ \forall a \in B$.

## Group - B

(10 Marks)

Answer any *two* questions : 5×2=10

2. If $R$ and $S$ be equivalence relations on the set $A$, prove that $R \cap S$ is an equivalence relation on the set $A$. 5

3. Use Karnaugh map to simplify $f(A, B, C) = A + BC + A'BC$. 5

4. Show that the following statement is a Tautology.

$$((P \vee Q) \wedge (P \to R) \wedge (Q \to R)) \to R.$$ 5

5. Construct the switching circuit representing $ab + ab' + a'b'$ and show that the circuit is equivalent to the switching circuit $a + b'$. 5

## Group - C

(18 Marks)

Answer any *two* questions : 9×2=18

6. (a) Prove that $K_5$, the complete graph of five vertices, is not planner graph. 5

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(b) Define an AND gate. Construct Block diagram symbol and Truth table for 4 input AND gate. 1+3

7. (a) Draw the undirected graph represented by the adjacency matrix $A$ given by

$$A=\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$ 5

(b) Transform the following Boolean expression in CNF into an expression in DNF :

$$(x+y'+z)(x+y+z')(x+y'+z')(x'+y+z)(x'+y+z').$$ 4

8. (a) Define Trees in graph theory. Prove that a tree $T$ with $n$ vertices has $n-1$ edges. 1+4

(b) Use truth table to prove the distributive law

$$p\vee(q\wedge r)\equiv(p\vee q)\wedge(p\vee r).$$ 4

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