2019

PHYSICS

(Honours)

Paper : PHYH-DC - 1

[CBCS]

Full Marks : 25 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *five* questions from the following : $2\times5=10$

(a) Show that the field $\vec{F}=\hat{i}\left(2xy+z^2\right)+\hat{j}x^2+\hat{k}(2xz)$ is conservative.

(b) Solve the following simultaneous equations :

$$\frac{dx}{dt}=-wy \text{ and } \frac{dy}{dt}=wx$$

(c) With the help of divergence theorem, show that

$$\int\left(\vec{\nabla}\phi\times\vec{\nabla}\psi\right)\cdot\vec{ds}=0$$

P.T.O.

(d) Evaluate the integral $I = \int_0^2 x^2 \delta(2x-1)\, dx$.

(e) Find out a unit vector perpendicular to the surface $x^2 + y^2 - z^2 = 11$ at the point (4, 2, 3).

(f) Comment on the homogeneity of the differential equation $aye^{x/y} dx + \left(y - axe^{x/y}\right) dy = 0$.

(g) Define polar and axial vectors. Give example of each.

(h) If $u(x, y) = \tan^{-1} \frac{y}{x}$, then find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.

2. Answer any *three* questions from the following : 5×3=15

(a) Verify Gauss's divergence theorem for the vector $\overline{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ w.r.t. a unit cube with two opposite corners at (0,0,0) and (1,1,1). 5

(b) (i) Given : $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, diagonalise A.

(ii) Find a similarity transformation matrix B which diagonalises A. 2+3=5

(c) Express $\vec{\nabla}$ in spherical polar coordinates and determine $\vec{\nabla}\psi(r,\theta,\phi)$ where $\psi(r,\theta,\phi) = 2r\sin\theta + r^2\cos\phi$. 3+2=5

(d) (i) Evaluate the integral $\int x^2 y\,dV$ where V is the closed region bounded by the planes $4x+2y+z=8$, $x=0$, $y=0$ and $z=0$.

(ii) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$ 3½+1½=5

(e) Solve the following equation :

$$\frac{d^2y}{dx^2}-y=x^2\cos(x)$$ 5
