U.G. 4th Semester Examinations 2022

MATHEMATICS (Honours)

Paper Code : DC-10

(Probability & Statistics)

[CBCS]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group-A

1. Answer any ***four*** questions : 1×4=4

(a) If two dice are thrown, what is the probability that the sum is greater than 9.

(b) A random variable X has a discrete set of values 0, 1, 2, 3 with corresponding probability mass distribution $\frac{1}{8}, \frac{3}{8}, \frac{1}{4}, \frac{1}{4}$ respectively. Find the distribution function of X.

(c) The distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0, & x \le 0 \\ x, & 0 < x \le 1 \\ 1, & x > 1 \end{cases}$$

Find the probability density function.

(d) Show that $E(cg(x)) = cE(g(x))$.

(e) $f(x) = \frac{4x}{5}$ when $0 < x \le 1$

$= \frac{2}{5}(3-x)$ when $1 < x \le 2$

$= 0$ elsewhere

Find $E(x)$.

(f) $f(x,y) = \dfrac{1}{(b-c)(d-c)}$ for $a < x < b, c < y < d$

$= 0$ elsewhere

is the Joint density function of a distribution of (x, y). Find the marginal distribution of y.

(g) A random variable x follows poisson distribution.

If $2P(X=2) = P(X=1) + 2P(X=0)$, then find the value of variance of X.

Group-B

Answer any ***two*** questions : 5×2=10

2. The Joint probability density function of X and Y is

$$f(x,y) = 8xy \text{ if } 0 \le x \le y,\ 0 \le y \le 1$$
$$= 0 \quad \text{elsewhere}$$

Examine whether X and Y are independent. Also compute Var X, Var Y.

3. The Joint probability distribution of two random variables X and Y are given by

$$P(X=0, Y=1) = \frac{1}{3},\ P(X=1, Y=-1) = \frac{1}{3},\ P(X=1, Y=1) = \frac{1}{3}$$

Find marginal distributions of X and Y.

4. Let (X, Y) have the general two-dimensional normal distribution and we make a linear transformations

$$U = (X - m_x)\cos\alpha + (Y - m_y)\sin\alpha$$

$$V = -(X - m_x)\sin\alpha + (Y - m_y)\cos\alpha$$

Show that U, V will be independent if $\tan(2\alpha) = \dfrac{2\rho\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}$, where $m_x, m_y, \sigma_x, \sigma_y, \rho$ have their usual meaning.

5. If $X(n, p)$ is a Binomial distribution, then prove that $\mu_{k+1} = p(1-p)\left(nk\mu_{k-1} + \dfrac{d\mu k}{dp}\right)$ where μ_r is the r^{th} central moment of the distribution.

Group-C

Answer any ***two*** questions : 9×2=18

6. (a) A random variable X has a density function $f(x)$ given by

$$f(x) = e^{-x}, \; x \geq 0$$
$$= 0, \text{ elsewhere.}$$

Show that Chebyshev's inequality gives $P(|x-1| \geq 2) \leq \frac{1}{4}$. 5

(b) The marks obtained by 17 students in an examination have a mean 57 and variance 64. Find 99% confidence interval for the mean of population of marks assuming it to be normal. [Given that $P(t > 2.921) = 0.005$ for 16 degrees of freedom]. 4

7. (a) If X and Y be two random variable such that $E(X^2)$, $E(Y^2)$ and $E(XY)$ exist, then prove that $\{E(XY)\}^2 \leq E(X^2)E(Y^2)$. Where the equality holds iff $E(X^2) = 0$ or $P(Y - aX = 0) = 1$ for some constant a. 4

(b) Prove that the maximum likelihood estimate of the Parameter α of a population having density function $\frac{2}{\alpha^2}(\alpha - x)$, $0 < x < \alpha$, for a sample of unit size is $2x$, x being the sample value. Show that the estimate is biased. 5

8. (a) The bivariate random variable (X, Y) jointly follow the probability density function

$$f(x, y) = kx^2(8 - y) \quad x < y < 2x, \; 0 \leq x \leq 2$$
$$= 0 \quad \text{elsewhere}$$

Find k and the conditional probability density functions $f_x^{(x/y)}$ and $f_y^{(y/x)}$. 5

(b) Let p denotes the probability of getting a head when a given coin is tossed once. Suppose that the hypothesis $H_0 : p = 0.5$ is rejected in favour of $H_1 : p = 0.6$ if 10 trails result in 7 or more heads, calculate the probability of type I and type II errors. 4