U.G. 5th Semester Examination 2021

MATHEMATICS (Honours)

Paper : DC-11

[Advanced Analysis on $\mathbb{R}$ & $\mathbb{C}$]

(CBCS)

Full Marks : 32 Time : 2 Hours

The figures in the margin indicate full marks.
Notations and symbols have their usual meanings.

Group - A

(4 Marks)

1. Answer any ***four*** questions : 4×1=4

(a) Let d be a metric on X. Determine all constants k such that kd is a metric on X.

(b) Give example of a sequence which is not convergent in a metric space.

(c) Show that the family $\left\{\left(\frac{1}{n},1\right): n \in \mathbb{N}, n \geq 2\right\}$ is an open cover of (0, 1).

(d) Let $f : D \to \mathbb{C}$ be an analytic function defined by

$f(z) = f(x+iy) = u(x,y) + iv(x,y), z = x+iy \in D$. Find $f'(z)$.

(e) Evaluate the line integral $\int_C \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the curve given by $z(t) = t^2 + it$.

(f) Show that the function f defined by $f(z) = \bar{z}, z \in \mathbb{C}$ is not analytic.

(g) Find the radius of convergence of $\sum \frac{zn}{\lfloor n}$.

Group - B

(10 Marks)

Answer any ***two*** questions : 2×5=10

2. State and prove Cantor's Intersection theorem. 5

3. Show that $C[a, b]$ with supremum metric is complete. 5

4. (a) Using Cauchy integral formula calculate the integral $\int_C \frac{z\,dz}{(9-z^2)(z+i)}$, where C is the circle $|z| = 2$ described in positive sense. 3

   (b) Expand $\frac{1}{z}$ as a Taylor's series about z = 1. 2

5. Let f be an analytic function on a region R. Show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

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Group - C

(18 Marks)

Answer any ***two*** questions : 2×9=18

6. (a) Let (X, d) be a metric space and A and B be two connected subsets of X such that $A \cap B \neq \emptyset$. Show that $A \cup B$ is a connected set of X. By an example, show that the intersection of two connected sets in a metric space need not be connected.

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   (b) Determine a conjugate harmonic function of the function $u = e^x\left(x\cos(y) - y\sin(y)\right)$ in the complex plane C. 5

7. (a) Let (X, d) be a metric space. Show that any convergent sequence in (X, d) is Cauchy sequence. Does the converse is hold? If not, give counter example. 3+2

(b) If $u = x^3 - 3xy^2$, show that there exists a functions $v(x, y)$ such that $w = u + iv$ is analytic in a finite region. 4

8. (a) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n+1}{n!} z^{n^3}$. 3

(b) Show that the mapping $f : \mathbb{C} \to \mathbb{C}$ defined by $f(z) = z^n$ is conformal at all points except $z = 0$. 2

(c) Find the Laurent's series which represents the function $\frac{z^2 - 1}{(z+1)(z+3)}$

when $2 < |z| < 3$. 4