P - III (1+1+1) H / 21(N)

2021

MATHEMATICS (Honours)

Paper Code : VIII - A & B

(New Syllabus)

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

  Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

  Subject Code : | III | A | & | B |

  Subject Name : [ ]

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

  Example — If alternative A of 1 is correct, then write :
  1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code : | III | A | & | B |

Subject Name : [ ]

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A) / (B) / (C) / (D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি **1** নম্বর প্রশ্নের সঠিক উত্তর **A** হয় তবে লিখতে হবে :

1. — A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Paper Code : VIII - A

Full Marks : 10 Time : Fifteen Minutes

Choose the correct answer.

Each question carries 2 marks.

Notations and symbols have their usual meanings.

1. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by $T(x, y, z) = (x + y, x - z)$. Then the dimension of the null space of T is

   A. 0

   B. 1

   C. 2

   D. 3

2. Let H and K be two normal subgroups of a group G with $H \subset K$. If $[G : H] = 10$ and $[G : K] = 5$, then $[K : H] =$

   A 5

   B 2

   C 10

   D 50

3. If the relation $B^i_j \; V_i = 0$ holds for any arbitrary covariant vector V_i, then

   A. $B^i_j = 0$

   B. $B^i_j = 1$

   C. $B^i_j = 2$

   D. none of these

4. The DNF (disjunctive normal form) of the Boolean function $a + ab$ is

   A. $b + ab$

   B. $ab + a'b'$

   C. $ab + a'b$

   D. $ab + ab'$

5. The Laplace transform of $e^{-3t}(2\cos 5t - 3\sin 5t)$ is

   A. $\frac{9s-2}{s^2-6s+34}$

   B. $\frac{2s-9}{s^2-6s+34}$

   C. $\frac{2s-9}{s^2+6s+34}$

   D. $\frac{9s-2}{s^2+6s+34}$

2021

MATHEMATICS (Honours)

Paper Code : VIII - B

(New Syllabus)

Full Marks : 50 Time : Two Hours Forty Five Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

1. Answer any *two* questions $4 \times 2 = 8$

   (a) If $T : U \to V$ is a linear transformation between two finite dimensional vector spaces U and V, then show that rank of T = rank of the matrix of T. 4

   (b) Show that a linear transformation $T : U \to V$ between two finite dimensional vector spaces U and V is non-singular if and only if T maps every linearly independent subset of U into a linearly independent subset of V. 4

   (c) Prove that the linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (ax + by, cx + dy)$ is invertible if $ad - bc \neq 0$. 4

2. Answer any *two* questions $3 \times 2 = 6$

   (a) If H is a subgroup of the commutative group G, then show that the quotient group G/H is commutative. 3

   (b) In a group G, prove that the subset $A = \{a \in G : ag = ga \ , \forall \ g \in G\}$ is a subgroup of G. Also prove that A is a normal subgroup of G. 1+2

   (c) If $f : (G, o) \to (G', *)$ is an isomorphism, then show that $f^{-1} : (G', *) \to (G, o)$ is also an isomorphism. 3

3. Answer any *two* questions $3 \times 2 = 6$

(a) Draw the circuit represented by the Boolean function $a(a' + b) + b(b + c) + b$ and simplify the function. 2+1

(b) Find the DNF and CNF of the Boolean function $f(x, y, z)$ such that $f(x, y, z) = 1$, if two of the variables are 1 and $f(x, y, z) = 0$ otherwise. 3

(c) In a Boolean algebra B, show that $ab + ab' + a'b + a'b' = 1$, for any $a, b \in B$. 3

4. Answer any *three* questions $5 \times 3 = 15$

(a) Find the Laplace transform of the following periodic function

$$f(t) = \begin{cases} t & \text{if } 0 < t < \pi \\ \pi - t & \text{if } \pi < t < 2\pi. \end{cases}$$ 5

(b) Find the inverse Laplace transform of $\frac{(s+2)^2}{(s^2+4s+8)^2}$. 5

(c) Find the Laplace transform of $\int\limits_0^t \frac{\sin x}{x}\, dx$. 5

(d) Using Laplace transform, solve $(D^2 - 3D + 2)y = 4t + 3e^t$, when $y(0) = 1$ and $y'(0) = -1$. 5

(e) Solve the equation $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$ in series, near the ordinary point $x = 0$. 5

5. Answer any *three* questions $5 \times 3 = 15$

(a) Show that the expression $A(i, j, k)$ is a tensor if its inner product with any arbitrary tensor B_r^{pq} is a tensor. 5

(b) Prove that the covariant derivative of the fundamental tensors g^{ij} and g_{ij} are zero. 5

(c) If A_i are the components of a covariant vector, then show that $\frac{\partial A_i}{\partial x^j}$ are not the components of a tensor but $\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ are the components of a tensor. 2+3

(d) If A^{ij} is a skew symmetric tensor, then show that

$$\frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}A^{ij})}{\partial x^i}$$

is a tensor. 5

(e) If θ is the angle between two non-null vectors u^i and v^i, then show that

$$\sin^2\theta = \frac{(g_{hj}g_{ik} - g_{hk}g_{ij})u^h u^j v^i v^k}{g_{hj}g_{ik}u^h u^j v^i v^k}.$$

Hence deduce that if u^i and v^i are orthogonal unit vectors, then $(g_{hj}g_{ki} - g_{hk}g_{ji})u^h v^i u^j v^k = 1.$ 3+2