2020

MATHEMATICS (General)

Paper : MTMG - DC-3/GE-3

[CBCS]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.

Notations and symbols have their usual meanings.

Group - A

1. Answer any **four** questions. $1 \times 4 = 4$

(a) Evaluate

$$\int_0^{\frac{\pi}{2}} (3 \sin u \ \mathbf{i} + 2 \cos u \ \mathbf{j}) du.$$

(b) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

(c) Write down the condition for maxima and minima for functions of two variables.

(d) Define linear span of a subset S of vector space.

(e) Define directional derivative for a function $f : \mathbb{R}^2 \to \mathbb{R}$.

(f) Write down the condition for three vectors to be coplanar.

(g) If $x = r\cos\theta,\ y = r\sin\theta$, then find the value of $\frac{\partial(x,y)}{\partial(r,\theta)}$.

Group - B

Answer any *two* questions. 5×2=10

2. Show that the points A, B, C whose position vectors are respectively $2\mathbf{i}+4\mathbf{j}-\mathbf{k}$, $4\mathbf{i}+5\mathbf{j}+\mathbf{k}$ and $3\mathbf{i}+6\mathbf{j}-3\mathbf{k}$ form a right angled triangle. [5]

3. (a) If $\phi = 2xz^4 - x^2y$, then find $|\nabla\phi|$ at the point $(2,-2,-1)$. [2]

   (b) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2,-1,2)$ in the direction $2\,\mathbf{i} - 3\,\mathbf{j} + 6\,\mathbf{k}$. [3]

4. State and prove Euler's theorem for a function of two variables. [5]

5. Find the work done in a moving particle in the force field $\mathbf{F} = 3x^2\,\mathbf{i} + (2xz-y)\,\mathbf{j} + z\,\mathbf{k}$ along the curve defined by $x^2 = 4y,\ 3x^3 = 8z$ from $x = 0$ to $x = 2$. [5]

Group - C

Answer any *two* questions. 9×2=18

6. (a) Verify Stoke's theorem for $\mathbf{A} = (2x-y)\,\mathbf{i} - yz^2\,\mathbf{j} - y^2z\,\mathbf{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2 = 1$ and C is its boundary. [5]

   (b) Evaluate

$$\iint_R (x^2+y^2)\;dx\;dy,$$

   where R is the region bounded by $y = x^2, x = 2, y = 1$. [4]

7. (a) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then prove that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$.

   [4]

   (b) Find the maximum or minimum value of $x^m y^n z^p$, subject to the condition

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1.$$ [5]

8. (a) Verify Green's theorem in the plane for

$$\oint_C (xy + y^2)dx + x^2 dy \ ,$$

where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. [5]

(b) Show that $\nabla r^4 = 4r^2 \ \mathbf{r}$. [4]