2020

MATHEMATICS (Honours)

Paper Code : III - A & B

[New Syllabus]

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

  Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

  Subject Code : | III | A | & | B |

  Subject Name : [ ]

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

  Example — If alternative A of 1 is correct, then write :
  1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

  উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

  Subject Code : | III | A | & | B |

  Subject Name : [ ]

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A) / (B) / (C) / (D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

  উদাহরণ — যদি **1** নম্বর প্রশ্নের সঠিক উত্তর **A** হয় তবে লিখতে হবে :

  1. — A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Paper Code : III - A

Full Marks : 20 Time : Thirty Minutes

Choose the correct answer.

Answer *all* the following questions,
each question carries 2 marks.

Notations and symbols have their usual meanings.

1. The point on $\frac{l}{r} = 1 - \cos\theta$ which has the smallest radius vector is

   (A) $(\frac{l}{2}, \pi)$

   (B) (l, π)

   (C) $(l, \frac{\pi}{2})$

   (D) $(\frac{l}{2}, -\pi)$.

2. The polar of the point $(0, 0)$ w.r.t. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is

   (A) $-gx + fy = 0$

   (B) $gx + fy = 0$

   (C) $gx + fy + c = 0$

   (D) $gx + fy - c = 0$.

3. The equation $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$ represents

   (A) an ellipse

   (B) a hyperbola

   (C) a pair of straight lines

   (D) a parabola.

4. The foot of the perpendicular from the origin to the plane is $(3, 2, -1)$. The equation of the plane is

   (A) $3x + 2y + z = 14$

   (B) $3x + 2y + z + 14 = 0$

   (C) $3x - 2y + z = 14$

   (D) $3x + 2y - z = 14$.

5. The volume of the tetrahedron formed by the points $(0,0,0)$, $(3,2,-1)$, $(4,1,4)$ and $(5,3,5)$ is

(A) $\frac{14}{3}$ cubic unit
(B) $\frac{14}{5}$ cubic unit
(C) $\frac{14}{11}$ cubic unit
(D) $\frac{14}{13}$ cubic unit.

6. If a right circular cone has three mutually perpendicular generators, then its semi-vertical angle is given by

(A) $\tan^{-1}(\sqrt{3})$
(B) $\tan^{-1}(\sqrt{2})$
(C) $\tan^{-1}(\sqrt{5})$
(D) $\tan^{-1}(\sqrt{7})$.

7. The velocity of a particle moving in a straight line at any instant t, when its distance from the origin is x, is given by $x = \frac{1}{2}v^2$. The acceleration of the particle is

(A) 2 unit
(B) -2 unit
(C) 1 unit
(D) 3 unit.

8. A particle describes the curve $r\cosh(n\theta) = a$ under a force F to the pole, the law of force is proportional to

(A) $1/r$
(B) $1/r^2$
(C) $1/r^3$
(D) none of the above.

9. A particle coming rest from infinity will reach the earth's surface with a velocity

(A) $\sqrt{gr}$
(B) $\sqrt{2gr}$
(C) $\sqrt{3gr}$
(D) $2\sqrt{gr}$.

10. A particle describes a parabola with uniform speed. The angular velocity of the particle about the focus S, at any point P, varies inversely as

(A) $(SP)^{5/2}$

(B) $(SP)^{7/2}$

(C) $(SP)^{3/2}$

(D) $(SP)^{9/2}$.

2020

MATHEMATICS (Honours)

Paper Code : III - B

[New Syllabus]

Full Marks : 80 Time : Three Hours Thirty Minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group-A
(20 Marks)

Answer any **four** questions.

$5\times 4=20$

1. If the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two straight lines equidistant from the origin, then show that $f^4-g^4=c(bf^2-ag^2)$.

2. Reduce the equation $11x^2+4xy+14y^2-26x-32y+23=0$ to the canonical form and state the nature of the conic.

3. Prove that the two conics $\frac{l_1}{r}=1-e_1\cos\theta$ and $\frac{l_2}{r}=1-e_2\cos(\theta-\alpha)$ will touch one another, if $l_1^2(1-e_2^2)+l_2^2(1-e_1^2)=2l_1l_2(1-e_1e_2\cos\alpha)$.

4. Find the locus of the poles with respect to the circle $x^2+y^2=a^2$ of the tangents to the circle $x^2+y^2=2ax$.

5. Show that the locus of the middle points of the normal chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2\cdot\left(\frac{a^6}{x^2}+\frac{b^6}{y^2}\right)=(a^2-b^2)^2$.

6. The origin is shifted to the point $(3,-1)$ and the axes are rotated through an angle $\tan^{-1}\dfrac{3}{4}$. If the co-ordinates of a point are $(5,10)$ in the new system, then find the co-ordinates in the old system.

Group-B
(25 Marks)
Answer any **five** questions.

5× 5=25

7. A point P moves on the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, which is fixed and the plane through P perpendicular to OP meets the axes in A, B, C. If the planes through A, B, C parallel to co-ordinate planes meet in a point Q, then show that the locus of Q is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}.$$

8. Show that the locus of the variable line which intersects the three lines $y=mx$, $z=c$; $y=-mx$, $z=-c$; $y=z$, $mx=-c$ is the surface $y^2-m^2x^2=z^2-c^2$.

9. A variable sphere passes through the points $(0,0,\pm c)$ and cuts the straight lines $y=x\tan\alpha$, $z=c$; $y=-x\tan\alpha$, $z=-c$ in the points P, P'. If $PP'=2a$, a constant, then show that the centre of the sphere lies on the circle $z=0$, $x^2+y^2=(a^2-c^2)\operatorname{cosec}^2 2\alpha$.

10. Show that the angle between the lines of section of the plane $x+y+z=0$ and the cone $\frac{yz}{b-c}+\frac{zx}{c-a}+\frac{xy}{a-b}=0$ is $60°$.

11. The section of the cone whose guiding curve is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $z=0$ by the plane $x=0$ is a rectangular hyperbola. Show that the locus of the vertex is the surface $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$.

12. Show that the equation of the right circular cylinder, whose guiding curve is the circle through the points $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ is $x^2+y^2+z^2-xy-yz-zx=1$.

13. Find the locus of a luminous point, if the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ casts a circular shadow on the plane $z=0$.

14. Show that the feet of the normals from the point (α,β,γ) to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie on the curve of intersection of the ellipsoid and the cone

$$\frac{\alpha a^2(b^2-c^2)}{x}+\frac{\beta b^2(c^2-a^2)}{y}+\frac{\gamma c^2(a^2-b^2)}{z}=0.$$

Group-C

(35 Marks)

Answer any **five** questions.

$7\times 5=35$

15. A particle moves from rest in a straight line under an attractive force $\mu\times(\text{distance})^{-2}$ per unit mass to a fixed point on the line. Show that if the initial distance from the centre of force is $2a$, then the distance will be a after a time $\left(\frac{\pi}{2}+1\right)\left(\frac{a^3}{\mu}\right)^{1/2}$.

16. Three elastic balls of masses m_1, m_2, m_3, lie in a straight line on a horizontal table and m_1 is projected towards m_2. If the velocity of m_1 after striking m_2 is equal to that of m_2 after striking m_3, then prove that

$$(m_1+m_2)(m_2+m_3)e = m_1m_3(1+e)^2.$$

17. Prove that the velocity and acceleration components referred to rotating axes are respectively

$$\dot{x}-y\dot{\theta},\quad \dot{y}+x\dot{\theta},\quad \ddot{x}-x\dot{\theta}^2-2\dot{y}\dot{\theta}-y\ddot{\theta},\quad \ddot{y}-y\dot{\theta}^2+2\dot{x}\dot{\theta}+x\ddot{\theta}\ .$$

18. A particle moves under a force $m\mu\{3au^4-2(a^2-b^2)u^5\}$, $(a>b)$, and is projected from an apse at a distance $a+b$ with velocity $\frac{\sqrt{\mu}}{a+b}$. Show that its orbit is $r=a+b\cos\theta$.

19. A particle descends down a rough circular tube starting from rest at an extremity of horizontal diameter. If it stops at the lowest point, then show that the coefficient of the friction satisfies the equation $3\mu e^{-\mu\pi}+2\mu^2-1=0$.

20. A heavy particle falls from rest under gravity in a medium, the resistance of which varies as the square of the velocity. Show that the depth x described in time t is given by $x=\frac{V^2}{g}\log\cosh\frac{gt}{V}$, where V is the terminal velocity.

21. If a rocket, originally of mass M, throws off every unit of time a mass eM with relative velocity V and if M' is the mass of the case etc., then show that it cannot rise at once unless $eV>g$, not at all unless $\frac{eMV}{M'}>g$. If it just rises vertically at once, then show that the greatest velocity is $V\log\frac{M}{M'}-\frac{g}{e}\left(1-\frac{M'}{M}\right)$.
