1st Unit Test Examination-2023

MATHEMATICS (Honours)

Paper Code: DC-H-13

Full Marks: 20 **Time: one Hour**

Group-A

Answer any four questions. $4 \times 5 = 20$

1. (a) State fundamental theorem of L.P.P.

   (b) An agricultural farm has 180 tons of Nitrogen fertilizers, 250 tons of phosphate and 220 tons of potash. It is able to sell 3 : 3 : 4 mixtures of these substances at a profit of Rs 15 per tons and 1 : 2 : 1 mixture at profit of Rs 12 per ton respectively. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit. [2+3=5]

2. Prove that in E^2, the set $X = \{(x, y) : y^2 \leq x\}$ is a convex set, while the set $X = \{(x, y) : y^2 \geq x\}$ is not a convex set. [5]

3. Prove that every extreme point of the convex set of all feasible solutions of the system

$$Ax = b, x \geq 0$$

   corresponds to a basic feasible solutions. [5]

4. If $x_1 = 2, x_2 = 3, x_3 = 1$ is a feasible solution of the L.P.P.
   Maximize $z = x_1 + 2x_2 + 4x_3$
   subject to

$$2x_1 + x_2 + 4x_3 = 11$$

$$3x_1 + x_2 + 5x_3 = 14$$

$$x_1, x_2, x_3 \geq 0$$

   Find a basic feasible solutions [5]

5. Solve the following problem by Big M-method
   Minimize $z = 2x_1 + 3x_2$
   subject to

$$2x_1 + 7x_2 \geq 22$$

$$x_1 + x_2 \geq 6$$

$$5x_1 + x_2 \geq 10$$

$$x_1, x_2 \geq 0$$

[5]

6. Maximize $z = 2x_1 + x_2$
   subject to

$$4x_1 + 3x_2 \leq 12$$

$$4x_1 + x_2 \leq 8$$

$$4x_1 - x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

   Find out the extreme points of the convex set of feasible solutions and hence find out the maximum value of the objective function. [5]

Gour Mahavidyalaya

MATHEMATICS (Honours)

Paper Code: MATH-H-DSE03(1)

Point set typology

Semister VI

Class test 1

Full Marks : 20 Time : 1 hour

1. Answer any one. 1×2=2

   (a) Define topology and topological space. Give example of a topology on $X = \{1, 2, 3\}$ and a collection of subsets of $X = \{1, 2, 3\}$ which is not a topology on X. [1+1]

   (b) What do you mean by lower limit topology and k-topology on real line. Which is finer? Justify. [1+1]

2. Answer any one. 1×3=3

   (a) Let $X = \{a, b, c, d, e\}$ and $\mathcal{A} = \{\{a, b, c\}, \{c, d\}, \{d, e\}\}$. Find the topology generated by $\mathcal{A}$.
   Is every basis is a subbasis and vice-versa? Justify. [2+1]

   (b) Let $\{\mathcal{T}_i : i \in \Lambda\}$ be any collection of topologies on X. Then the intersection $\bigcap \mathcal{T}_i$ forms a topology. Is also $\bigcup \mathcal{T}_i$ forms a topology? Justify. Where Λ is index set. [2+1]

3. Answer any one. 1×5=5

   (a) Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each $x \in$ U there is an element C of $\mathcal{C}$ such that $x \in$ C $\subset$U. Then show that $\mathcal{C}$ is a basis for that topology of X. [5]

   (b) Consider $Y = [-3, 3]$ as a subspace of $\mathbb{R}$. Which of the following sets are open in Y ? Which are open in $\mathbb{R}$? Justify with proper explanation.
   $A = \{x \in \mathbb{R} : 2 < |x| \leq 3\}$
   $B = \{x \in \mathbb{R} : -2 < x \leq 3 \text{ and } \frac{x}{3} \notin \sin^{-1}\frac{n\pi}{4}; n = 1, 2, 3, 4.\}$
   $C = \{x \in \mathbb{R} : 2 \leq |x| < 3\}$ [5]

(Turn over)

4. Answer any two. $2 \times 2 = 4$

   (a) Let A and B be two connected subspace of X such that A∩B$\neq \phi$. Does it imply A∩B is connected? Justify your answer. [2]

   (b) Let $\mathcal{T}_1$ and $\mathcal{T}_1$ are two topologies on X. If $\mathcal{T}_1 \subseteq \mathcal{T}_2$, what does disconnectedness of X in one topology imply about disconnectedness in other? [2]

   (c) Is the subspace $Y = [-1, 0) \cup (0, 1]$ connected in the subspace topology of $\mathbb{R}$. Justify. [2]

5. Show that the closure of a connected set is connected. [3]

6. Prove that the only connected subsets of $\mathbb{Q}$ are singleton sets, in the subspace topology of $\mathbb{R}$ [3]

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