1^{ST} Internal Exam/UG/5th Sem/H/22 (CBCS)/GM

2022

GOUR MAHAVIDYALAYA

DEPARTMENT OF MATHEMATICS

Paper : MTMH - DC-11

[CBCS]

Full Marks: 15 Time: 45 minutes

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

1. Answers any two: [2×5=10]

(a) Let A be the set of all sequences of complex numbers. Consider a function $\rho : A \times A \to \mathbb{R}$ given by $\rho(x, y) = \sum\limits_{n\in\,\mathbb{N}} a^n \frac{|x_n - y_n|}{1+|x_n-y_n|}$, where $x = \{x_n\}$, $y = \{y_n\} \in A$ and $\sum\limits_{n\in\,\mathbb{N}} a_n$ is any convergent series of positive terms. Prove that (A, ρ) is a metric space. [5]

(b) State and prove Hausdroff property for a metric space. [1+4]

(c) Let us define a mapping $d : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ by

$$d(x, y) = \begin{cases} |x_1 - y_1| & \text{if } x_2 = y_2, \\ |x_2 - y_2| + |x_1| + |y_1| & \text{if } x_2 \neq y_2, \end{cases}$$

where $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$. Examine whether d is a metric or not. [5]

2. (i) Let (X, ρ) be a metric space. Then for all x, y, $z \in X$ prove that $|\rho(x, u) - \rho(y, u)| \leq \rho(x, y)$.

(ii) Show by an example that intersection of infinitely many open sets need not to be open in a metric space.

(iii) Let A be a non-empty set in a metric space (X, d). Prove that $Int(A)$ is an open set. [2+1+2]

GOUR MAHAVIDYALAYA
Deparment of Mathematics
5th Sem-1st Class test
Paper Code-DC12

Time: 45 Minutes F.M.:15

1. Answer any three questions. 5× 3=15

   (a) Explain Gauss-Elimination Method to solve a system of linear equation containing n number of variables. [5]

   (b) Evaluate $\int_0^{0.5} \frac{x}{cosx} dx$ by Simpson's One-third Rule, taking $n = 10$. [5]

   (c) Solve the following system of equations

   $$3x_1 + 2x_2 - 4x_3 = 12$$
   $$-x_1 + 5x_2 + 2x_3 = 1$$
   $$2x_1 - 3x_2 + 4x_3 = -3$$

   by matrix factorisation(LU-decomposition method) method or Gauss-Seidle method. [5]

   (d) From the following table find the value of $f(2.5)$:

x	2	3	5	7
f(x)	0.301	0.477	0.699	0.845

   using Newton's divided difference formula. [5]

   (e) Explain the method of Lagrange's Interpolation Formula for a given set of $(n + 1)$ arguments. [5]

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Gour Mahavidyalaya

MATHEMATICS (Honours)

Paper Code: MATH-H-DSE01(1)

Semister V

Class test 1

Full Marks : 15 Time : 45 minutes

1. Answer any two questions.
   (a) Let G be a group. Prove that $Aut(G)$ is a subgroup of $A(G)$. Where $Aut(G)$ denote the set of all automorphism of G and $A(G)$ denote the group of all permutation of G. [5]
   (b) Show that the set $Inn(G)$ of all inner automorphism of G is a normal subgroup of $Aut(G)$. [5]
   (c) Let G be a group. Show that $G/Z(G) \cong \text{Inn}(G)$ [5]
   (d) Let G be an infinite cycle group. Find $Aut(G)$. [5]
   (e) Let G be a group of order 4. Discuss about $Aut(G)$.
   Show that $Inn(G) = \{e\}$ if G is abelian. [2+3]
   (f) Find upto isomorphism $Aut(\mathbf{Z}_{72})$, $Aut(U(8))$, $Aut(\mathbf{Z})$, $Aut(\mathbf{Z}_2 \times \mathbf{Z}_2)$, $Inn(\mathbf{Q}_8)$, $Inn(S_3)$.
   Given $Aut(G_1) \cong Aut(G_2)$. Is $G_1 \cong G_2$ is true,in general? if not give an example. [3+2]
2. Answeer any one.
   (a) Define characterstic subgroup.
   $\text{H} = \{\bar{0}, \bar{2}\}$ is a subgroup of $\mathbf{Z}_4$. Show that H *char* $\mathbf{Z}_4$
   Show that $Z(G)$ *char* G. [1+2+2]
   (b) Show that every subgroup of a cyclic group (finite or infinite) is a characterstic subgroup. [5]
   (c) Show that a characterstic subgroup of G is a normal subgroup of G. Is the converse true? [5]
   [Hint: for converse $G = K_4 = \{e, a, b, c\}$ *and* $H = \{e, a\}$*]*

TUTORIAL/UG/5th Sem/H/22/GM (CBCS)

Gour Mahavidyalaya

MATHEMATICS (Honours)

Paper Code: MATH-H-DSE01(1)

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GOUR MAHAVIDYALAYA

B.Sc. 5th Semester(Honours)CBCS

Unit test-I

Subject: Mathematics

Paper:DSE-H-02

Full Marks : 15 Time : 45 minutes

Group-A

Answer any one of the following questions. [1 × 15 = 15]

1. (a) Explain the term "perfect fluid" with example. [2]

   (b) Prove that if the fluid is in equilibrium the pressure at a point is the same in every direction. Is it is true if the fluid is in motion? [5]

   (c) A fine glass tube in the shape of an equilateral triangle is filled with equal volume of three liquids which does not mix, whose densities are in arithmetical progression. The tube is held in a vertical plane, and the side that contains portions of the heaviest and lightest liquids makes an angle θ with the vertical. show that the surface of separation divide the side in the ratio

$$\cos\left(\frac{\pi}{6}-\theta\right) : \cos\left(\frac{\pi}{6}+\theta\right)$$

   . [8]

2. (a) Define pressure at a point in a fluid . [3]

   (b) Show that in a homogeneous fluid at rest under gravity, the difference of the pressures between any two points in proportional to the difference of their depths [5]

   (c) A semi-circular tube has its bounding diameter horizontal and contains equal volumes of n fluids of densities successively equal to $\rho, 2\rho, 3\rho, \ldots$,arranged in this order, If each fluid subtends an angle 2α at the centre and the tube just holds them all, show that

$$\tan n\alpha = (2n+1)\tan\alpha$$

   . [7]

GOUR MAHAVIDYALAYA
Deparment of Mathematics
5th Sem-1st Class test
Paper Code-SEC1

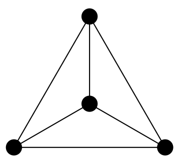
Time: 45 Minutes F.M.:15

1. Answer all the questions. . 5

   (a) Define subgraph of a graph with suitable exampe.

   (b) Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

   (c) Explain whether the following graph is a bipartite graph or not.



[1+2+2]

2. Answer any two questions. $5\times 2=10$

   (a) Define Hamiltonian Circuit. Prove that in a connected graph with n vertices there are $(n-1)/2$ edge-disjoint Hamiltonian circuits, if n is odd number≥ 3.. [1+4]

   (b) Prove that a given graph G is an Euler graph iff all vertices of G are of even degree.. [5]

   (c) What do you mean by a component of a graph? Prove that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges. [1+4]

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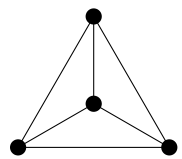

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