GOUR MAHAVIDYALAYA 1ST UNIT TEST

MATHEMATICS (Honours)

Paper Code: DC-H-13

Full Marks: 20 **Time: One Hour**

Group-A

(Marks 20)

Answer any four questions. $4 \times 5 = 20$

1. Define Convex set, Convex hull and Convex polyhedron in E^n. If x_1, x_2 be real number.
   Show that the set $X = \{(x_1, x_2) : 9{x_1}^2 + 4{x_2}^2 \leq 36\}$ is a convex set. [5]
2. Define Extreme point of a set and .Find the extreme point, if any of the following sets. [5]
   (a) $S = \{(x, y) : x^2 + y^2 \leq 16\}$
   (b) $P = \{(x, y) : |x| \leq 1, |y| \leq 1\}$.
   (c) $W = \{(x, y) : x^2 + y^2 \leq 1, x \geq 0, y \geq 0\}$.
3. State fundamental theorem of L.P.P.
   If $x_1 = 2, x_2 = 3, x_3 = 1$ is a feasible solution of the L.P.P.

   $$\text{Maximize } z = x_1 + 2x_2 + 4x_3$$

   Subject to

   $$\begin{aligned} 2x_1 + x_2 + 4x_3 &= 11 \\ 3x_1 + x_2 + 5x_3 &= 14 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

   find a basis feasible solution. [5]
4. Find the optimal solution to the following transportation problem: [5]

	W_1	W_2	W_3	W_4	
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
	5	8	7	14	

5. Find the optimal assignment to find the minimum cost for the assignment problem with the following matrix [5]

	I	II	III	IV	V
A	2	9	2	7	1
B	6	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

Class test - 1/UG/6th Sem/H/22/GM (CBCS)

2022
Gour Mahavidyalaya

MATHEMATICS (Honours)

Paper Code: MATH-H-DSE 03

Point set topology

[CBCS]

Full Marks : 20 Time : 1 hr 15 minutes

The figures in the margin indicate full marks.
Notations and symbols have their usual meanings.

Answer any two questions from 1 to 4. 5×2=10

1. Define subbasis for a topology on X. Give example.
   Let $X = \{a, b, c, d, e\}$ and let $A = \{\{a, b, c\}, \{c, d\}, \{d, e\}\}$. Find the topology on X generated by A. [5]

2. When two topologies on X are said to be comparable?
   The topologies of $\mathbb{R}_l$ (lower limit topology) and $\mathbb{R}_k$ (K-topology) are not comparable. [5]

3. If $\{\tau_\alpha : \alpha \in I\}$ be a family of topologies on X, show that $\bigcap\limits_{\alpha\in I} \tau_\alpha$ is a topology on X
   Is $\bigcup\limits_{\alpha\in I} \tau_\alpha$ a topology on X? Justify. [5]

4. Consider the set $Y = [-1, 1]$ as a subspace of R. Which of the following sets are open in Y? Which are open in R?
   $$A = \{x : \tfrac{1}{2} < |x| < 1\}$$
   $$B = \{x : \tfrac{1}{2} < |x| \le 1\}$$
   $$C = \{x : \tfrac{1}{2} \le |x| < 1\}$$
   $$D = \{x : \tfrac{1}{2} \le |x| \le 1\}$$
   Discuss. [5]

5. Answer any two questions. 4×2=8
   (a) Prove that only connected subsets of $\mathbb{Q}$ are singletons in the subspace topology of $\mathbb{R}$. [4]
   (b) Prove that connectedness is a topological property. [4]
   (c) Let X and Y be two connected topological spaces. Show that the product space $X \times Y$ is also connected. [4]

6. Answer any one question. 2×1=2
   (a) Show that $\mathbb{R}$ is connected in cofinite topology. [2]
   (b) Let τ and τ' are topological spaces on X. If $\tau' \supset \tau$ What does connectedness of X in topology imply about connectedness in the other? [2]