2022

GOUR MAHAVIDYALAYA
DEPARTMENT OF MATHEMATICS
Paper : MTMH - DC-03
[CBCS]

The figures in the margin indicate full marks.
Notations and symbols have their usual meanings.

Group- A

[Full marks: 20]

1. Answer any one question: [1×5=5]

(a) State and prove the Archimedean property of $\mathbb{R}$. [1+4]

(b) (i) Let A be a subset of $\mathbb{R}$. Show that $(A')' \subset A'$. [3]

(ii) Let $A, B \subset \mathbb{R}$. Is $(A \cap B)' = A' \cap B'$? Justify your answer. [1+1]

2. State density property of $\mathbb{R}$. Show that there does not exist any rational number r satisfying $r^2 = 2$. Find the derived set of the set $T = \left\{\frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N}\right\}$. [1+2+2]

Group- B

3. Answer any one question: [1×4=4]

(a) Prove that the sequences $\{x_n\}$ and $\{y_n\}$ defined by $x_{n+1} = \frac{1}{2}(x_n + y_n)$, $\frac{2}{y_{n+1}} = \frac{1}{x_n} + \frac{1}{y_n}$ for $n \geq 1$. $x_1, y_1 > 0$ converges to a common limit l, where $l^2 = x_1 y_1$. [4]

(b) Prove that the sequence $\{x_n\}$ defined by $x_1 = \sqrt{2}$ and $x_{n+1} = \sqrt{2x_n}$ $\forall n \geq 1$ converges to 2. [4]

4. Show that $\lim\limits_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right) = 1$. [3]

5. What is null sequence? Show that $\{\frac{n!}{n^n}\}$ is a null sequence. [1+2]

GOURMAHAVIDYALAYA 1ST UNIT TEST

MATHEMATICS (Honours)

Paper Code: DC-H-04

Full Marks: 20 **Time: one Hour**

Notations and symbols have their usual meanings

Group-A

(Marks 12)

1. Answer any four questions. $3 \times 4 = 12$

(a) Define the idempotent element in a group. Find the idempotent element in the monoid $(\mathbb{Z}_5, \cdot)$ and $(\mathbb{Z}_6, \cdot)$ [1+2=3]

(b) Let $(S, \circ)$ be a semigroup. If for $x, y \in S$, $x^2 \circ y = y = y \circ x^2$. Prove that $(S, \circ)$ as an abelian group. [3]

(c) Prove that the group $(G, \circ)$ is abelian if and only if $(a \circ b)^{-1} = a^{-1} \circ b^{-1}$ for all $a, b \in G$. [3]

(d) What are the difference between Symmetric group S_3 and Klein's 4-group V ? Given your answer with justifications. [3]

(e) Examine if the ring of matrices $\left\{ \begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}$ contains divisors of zero.[3]

Group-B

(Marks 8)

Answer any two questions. $2 \times 4 = 8$

2. Define characteristic of a ring. Show that the characteristic of an integral domain is either zero or prime number. [4]

3. Show that a finite integral domain is a field. [4]

4. Find the units element in the ring $(\mathbb{Z}_{10}, +, \cdot)$. Prove that the units form a cyclic group under multiplication. [4]