UG/3rd Sem/H/20 (CBCS)

2020

MATHEMATICS (Honours)

Paper : MTMH - DC- 07

[CBCS]

Full Marks : 32 | Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.

Notations and symbols have their usual meanings.

Group - A

1. Answer any **four** questions. $1 \times 4 = 4$

(a) State Young's theorem for a function of two variables.

(b) Examine the existence of the unique implicit function near the point $(1,0)$ for the equation $x^2 + y^2 = 1$.

(c) Find the value of

$$\iint_R xy \, dxdy \, ,$$

where

$$R = \left\{(x,y) : 0 \le x \le \frac{\pi}{2}, 0 \le y \le \frac{\pi}{2}\right\}.$$

(d) Change the order of the integration

$$\int_0^1 dy \int_0^y f(x,y) dx.$$

(e) Find a vector $\boldsymbol{\alpha}$ which is perpendicular to each of $6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$.

(f) If $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, show that $\nabla r^2 = 2\mathbf{r}$.

(g) Prove that the vector $3y^4z^2\mathbf{i} + 4x^3z^2\mathbf{j} - 3x^2y^2\mathbf{k}$ is solenoidal.

Group - B

Answer any *two* questions. 5×2=10

2. Determine whether the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = \sqrt{|xy|}$ is differentiable at $(0, 0)$. [5]

3. Evaluate
$$\int_0^1 dx \int_0^x \sqrt{x^2 + y^2}\,dy.$$ [5]

4. Verify Stokes' theorem for the vector field $\mathbf{A} = (2x - y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary of S. [5]

5. If $\mathbf{F}$ is a conservative vector field, then prove that $\mathbf{Curl}\,\mathbf{F} = \mathbf{0}$. [5]

Group - C

Answer any *two* questions. 9×2=18

6. (a) Suppose that H is a homogeneous function of degree n in the variables x and y having continuous first order partial derivatives. If $u(x, y) = (x^2 + y^2)^{-\frac{n}{2}}$, then prove that
$$\frac{\partial}{\partial x}\left(H\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(H\frac{\partial u}{\partial y}\right) = 0.$$ [6]

(b) Evaluate
$$\int_0^{\frac{\pi}{2}} dx \int_0^{\cos x} x^2\,dy.$$ [3]

7. (a) If $\mathbf{F} = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$, then evaluate $\iint\limits_S \mathbf{F} \cdot \mathbf{n} \, dS$, where S is the surface of the cube bounded by $x = 0, x = 1; y = 0, y = 1; z = 0, z = 1$. [5]

(b) Prove that the function $f(x, y) = \begin{cases} \frac{2xy}{x^2+y^2} & \text{for} \quad x^2 + y^2 \neq 0 \\ 0 & \text{for} \quad x^2 + y^2 = 0 \end{cases}$

is continuous in each variable separately. Also determine whether f is continuous at $(0, 0)$. [3+1]

8. (a) Divide the number 120 into three parts such that sum of their products taken two at a time is a maximum. [3]

(b) Find the directional derivative of $\psi(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. [3]

(c) For a scalar field ϕ, prove that $\textbf{Curl}(\textbf{grad}\, \phi) = \mathbf{0}$. [3]