

2020

MATHEMATICS (Honours)

Paper Code : II - A & B

[New Syllabus]

Important Instructions for Multiple Choice Question (MCQ)

- Write Subject Name and Code, Registration number, Session and Roll number in the space provided on the Answer Script.

Example : Such as for Paper III-A (MCQ) and III-B (Descriptive).

Subject Code :

III	A	&	B
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Subject Name :

- Candidates are required to attempt all questions (MCQ). Below each question, four alternatives are given [i.e. (A), (B), (C), (D)]. Only one of these alternatives is 'CORRECT' answer. The candidate has to write the Correct Alternative [i.e. (A)/(B)/(C)/(D)] against each Question No. in the Answer Script.

Example — If alternative A of 1 is correct, then write :

1. — A

- There is no negative marking for wrong answer.

মাল্টিপল চয়েস প্রশ্নের (MCQ) জন্য জরুরী নির্দেশাবলী

- উত্তরপত্রে নির্দেশিত স্থানে বিষয়ের (Subject) নাম এবং কোড, রেজিস্ট্রেশন নম্বর, সেশন এবং রোল নম্বর লিখতে হবে।

উদাহরণ — যেমন Paper III-A (MCQ) এবং III-B (Descriptive)।

Subject Code :

III	A	&	B
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Subject Name :

- পরীক্ষার্থীদের সবগুলি প্রশ্নের (MCQ) উত্তর দিতে হবে। প্রতিটি প্রশ্নে চারটি করে সম্ভাব্য উত্তর, যথাক্রমে (A), (B), (C) এবং (D) করে দেওয়া আছে। পরীক্ষার্থীকে তার উত্তরের স্বপক্ষে (A)/(B)/(C)/(D) সঠিক বিকল্পটিকে প্রশ্ন নম্বর উল্লেখসহ উত্তরপত্রে লিখতে হবে।

উদাহরণ — যদি 1 নম্বর প্রশ্নের সঠিক উত্তর A হয় তবে লিখতে হবে :

1. – A

- ভুল উত্তরের জন্য কোন নেগেটিভ মার্কিং নেই।

Paper Code : II - A

Full Marks : 20

Time : Thirty Minutes

Choose the correct answer.

Answer *all* the following questions,
each question carries 2 marks.

Notations and symbols have their usual meanings.

1. Interior of the set \mathbb{Z} is

- A. \emptyset
- B. \mathbb{Z}
- C. \mathbb{R}
- D. None of these.

2. The set $\{\frac{1}{n} : n \in \mathbb{N}\}$ is

- A. open in \mathbb{R}
- B. dense in \mathbb{R}
- C. closed in \mathbb{R}
- D. None of these.

3. $\lim_{n \rightarrow \infty} (1 + \frac{1}{3n})^n$ is

- A. e
- B. e^3
- C. $e^{\frac{1}{3}}$
- D. None of these.

4. Let $u_n = \frac{1}{n}$ and $v_n = \frac{(-1)^n}{n}$, where $n \in \mathbb{N}$. Then which of the following is true?
- A. $\sum_{n=1}^{\infty} u_n$ is convergent and $\sum_{n=1}^{\infty} v_n$ is divergent
 - B. $\sum_{n=1}^{\infty} u_n$ is divergent and $\sum_{n=1}^{\infty} v_n$ is convergent
 - C. both the series $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ are convergent
 - D. both the series $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ are divergent.
5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function. Which of the following is always true?
- A. f is bounded
 - B. f maps interval to interval
 - C. f is uniformly continuous
 - D. f maps open set to open set.
6. Suppose a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is such that $|f(x) - f(y)| \leq (x - y)^4$, for all $x, y \in \mathbb{R}$. Then for any $x \in \mathbb{R}$, the value of $f'(x)$ is
- A. 0
 - B. 1
 - C. 2
 - D. 4.
7. The expression $f(x) = e + e(x - 1) + \frac{e}{2!}(x - 1)^2 + \frac{e}{3!}(x - 1)^3 + \dots$ is the Taylor series for
- A. $f(x) = x$ about $x = e$
 - B. $f(x) = x^2$ about $x = -1$
 - C. $f(x) = x^2$ about $x = 1$
 - D. $f(x) = e^x$ about $x = 1$.

8. The value of $\int_0^{\infty} e^{-x^2} dx$ is

- A. 0
- B. $\sqrt{\pi}$
- C. $\frac{\sqrt{\pi}}{2}$
- D. None of these.

9. The value of $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6}$

- A. is 0
- B. is 1
- C. is -1
- D. does not exist.

10. Radius of curvature of the curve $y = 4 \sin x - \sin 2x$ at $(\frac{\pi}{2}, 4)$ is

- A. $\frac{5\sqrt{5}}{4}$
 - B. $\frac{5\sqrt{5}}{2}$
 - C. $5\sqrt{5}$
 - D. 1.
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2020

MATHEMATICS (Honours)**Paper Code : II - B****[New Syllabus]**

Full Marks : 80

Time : Three Hours Thirty Minutes

*The figures in the margin indicate full marks.**Notations and symbols have their usual meanings.***Group-A
(35 Marks)**Answer any **seven** questions $5 \times 7 = 35$

1. Prove that for any positive real number x , there exists a natural number n such that $0 < \frac{1}{n} < x$.
Find the derived set of the set $A = \{m + \frac{1}{n} : m, n \in \mathbb{N}\}$. [3+2]
2. Prove that the sequence $\{x_n\}$ defined by
 $x_1 = \sqrt{6}$,
 $x_{n+1} = \sqrt{6 + x_n}$, for $n \geq 1$
is convergent. Hence find the limit. [3+2]
3. Show that the sequence $\{x_n\}$ defined by $x_n = (1 + \frac{1}{n})^n$ is convergent and $\lim_{n \rightarrow \infty} x_n$ lies between 2 and 3. [5]
4. Prove that an absolutely convergent series is convergent. Hence test the convergence of the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$. [3+2]
5. Give definition of conditionally convergent series. Test the convergence of the series $\frac{(\log 2)^2}{2^2} + \frac{(\log 3)^2}{3^2} + \frac{(\log 4)^2}{4^2} + \dots + \frac{(\log n)^2}{n^2} + \dots$. [1+4]

6. Let f, g be defined on $A \subseteq \mathbb{R}$ to \mathbb{R} , and let c be a cluster point of A . Suppose that f is bounded on a neighbourhood of c and that $\lim_{x \rightarrow c} g(x) = 0$. Prove that $\lim_{x \rightarrow c} f(x)g(x) = 0$.

Hence show that $\lim_{x \rightarrow 0} x^2 \cos \frac{2}{x} = 0$. [3+2]

7. A real valued function f is continuous on $[0, 2]$ and $f(0) = f(2)$. Prove that there exists at least one point c in $[0, 1]$ such that $f(c) = f(c + 1)$. [5]

8. Prove that the equation $(x - 1)^3 + (x - 2)^3 + (x - 3)^3 + (x - 4)^3 = 0$ has only one real root. [5]

9. (a) Evaluate $\lim_{x \rightarrow 0} (\cos x)^{\cot^2 x}$.

- (b) A real valued function f is defined on some neighbourhood of a and f is differentiable at a . Prove that $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a-h)}{2h} = f'(a)$. [3+2]

10. Prove that $0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1$, for $x > 0$. [5]

Group-B
(20 Marks)

Answer any **four** questions

5 × 4 = 20

11. Use ϵ - δ definition to show that the function

$$f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is continuous at the origin. [5]

12. Let

$$f(x, y) = \begin{cases} y \sin \frac{1}{x} + \frac{xy}{x^2 + y^2}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Verify that $\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y)$ exists but neither $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ nor $\lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x, y)$ exists.

[1+3+1]

13. Determine whether the function $f(x, y) = \sqrt{|xy|}$ is differentiable at the origin. [5]

14. If $\frac{x^2}{a+u} + \frac{y^2}{b+u} + \frac{z^2}{c+u} = 1$, then show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right).$$

15. If [5]

$$\begin{aligned} u^3 + v + w &= x + y^2 + z^2, \\ u + v^3 + w &= x^2 + y + z^2 \\ \text{and } u + v + w^3 &= x^2 + y^2 + z, \end{aligned}$$

then prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{1 - 4(xy + yz + zx) + 16xyz}{2 - 3(u^2 + v^2 + w^2) + 27u^2v^2w^2}.$$

[5]

16. Let $u = f(x, y)$, where $x = r \cos \theta$ and $y = r \sin \theta$. Prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2}.$$

[5]

Group-C
(25 Marks)

Answer any **five** questions

5 × 5 = 25

17. Find the volume of the solid formed by revolving the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x -axis. [5]

18. Show that the radius of curvature of the envelope of the line $x \cos \theta + y \sin \theta = f(\theta)$ is $f(\theta) + f''(\theta)$. [5]

19. Find the asymptotes of the polar curve $r = \frac{a\theta}{\theta - 1}$. [5]

20. Find the area included between the curves $y^2 = 4a(x + a)$ and $y^2 = 4b(b - x)$. [5]

21. Evaluate:

$$\int_0^1 \frac{dx}{(1 - x^6)^{\frac{1}{6}}}$$

[5]

22. Find the centre of gravity of the area in the first quadrant bounded by the parabola $y^2 = 4ax$, the latus rectum and the x -axis. [5]

23. Find the pedal equation of the cardioid $r = a(1 + \cos \theta)$. [5]
