UG 1st Semester Examination 2021

MATHEMATICS (Honours/General)

Paper : DC-1 / GE-1

[Classical Algebra & Analytic Geometry]

(CBCS)

Full Marks : 32 Time : 2 Hours

The figures in the margin indicate full marks.
Notations and symbols have their usual meanings.

Group - A

(4 Marks)

1. Answer any ***four*** questions : 4×1=4

(a) Find the value of $\phi(323)$.

(b) Let A be a skew-symmetric matrix of order 3. What is the value of *det(A)*.

(c) Find the modulus and argument of $-1-i$.

(d) Apply Descartes' rule of signs to determine the minimum number of complex roots of the equation : $x^7 - 3x^3 + 1 = 0$.

(e) Find the points on the x-axis whose distance from the point (α, β, γ) is $\sqrt{\alpha^2 + \beta^2 + \gamma^2}$.

(f) Find the center and radius of the sphere $x^2 + y^2 + z^2 + 2x + 2y + 2z - 12 = 0$.

(g) Determine the rank of the matrix : $\begin{pmatrix} 0 & 2 & 1 & 3 \\ 2 & 0 & 3 & 0 \\ 1 & 3 & 0 & 1 \end{pmatrix}$.

Group - B

(10 Marks)

Answer any ***two*** questions : 2×5=10

2. Use the principle of induction to prove that $\left(3+\sqrt{5}\right)^n+\left(3-\sqrt{5}\right)^n$ is divisible by 2^n, for all $n \in \mathbb{N}$.

3. Use Laplace's expansion to prove that $\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = \left(a^2+b^2+c^2+d^2\right)^2$. 5

4. A change of the rectangular axes, without changing the origin, transforms $ax^2+2hxy+by^2$ and $cx^2+2gxy+dy^2$ to $a'x'^2+2h'x'y'+b'y'^2$ and $c'x'^2+2g'x'y'+d'y'^2$, respectively. Show that $ad+bc-2hg=a'd'+b'c'-2h'g'$. 5

5. Show that only one tangent plane can be drawn to the sphere

   $x^2+y^2+z^2-2x+6y+2z+8=0$ through the line $3x-4y-8=0=y-3z+2$. 5

Group - C

(18 Marks)

Answer any ***two*** questions : 2×9=18

6. (a) If α,β,γ be the roots of the equation $x^3+qx-r=0$, then find the value of $\sum\frac{1}{\alpha^2-\beta\gamma}$. 5

   (b) Prove that $3\cdot4^{n+1}\equiv 3\left(\bmod 9\right)$, where $n \in \mathbb{N}$. 4

7. (a) If $\tan\left(\theta+i\phi\right)=\sin\left(\alpha+i\beta\right)$, prove that $\sin 2\theta\cot\alpha=\sin h\,2\phi\cot h\,\beta$. 5

(b) If A be the matrix $\begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$ then show that $A^2 - 10A + 16I = O$. Hence obtain A^{-1}. 4

8. (a) Find the values of a and b for which the plane $ax + by + 5z - 7 = 0$ is perpendicular to the line $x = 4r + 3,\ y = -5r + 4, z = -4r - 2$, where r is a parameter. 4

(b) A conic Γ' is described having the same focus and eccentricity as the conic $\Gamma' : \frac{l}{r} = 1 + e\cos\theta\left(e < 1\right)$. The two conics Γ and Γ' touch each other only at the point θ with θ = α. Prove that the latus rectum of the conic Γ' is $\frac{2l\left(1 - e^2\right)}{1 + 2e\cos\alpha + e^2}$.

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